BM20A8800 Discrete Models and Methods 3op

Exercise 1 / Week 3

1. Formulate truth tables for following expressions. (At least in b and c, please use also some midterm columns.)

a) $\neg p \wedge q$ b) $p \vee q \Rightarrow p \wedge q$ c) $p \vee (q \Rightarrow r)$

2. Peirce's arrow (also known as Nicod's function) $\downarrow$ is defined as "not _ nor _", and its truth table is presented on the right. Come up with a proposition that uses ONLY Peirce's arrows as connectives (one or several arrows, but nothing else – not even negations!) and produces the same truth values as the original proposition.

p	q	$p \downarrow q$
0	0	1
0	1	0
1	0	0
1	1	0

a) $\neg p$ b) $p \wedge q$ c) $p \vee q$

3. Is the proposition $\neg(\neg(p \Rightarrow \neg q) \vee \neg(q \Rightarrow \neg p))$ true, if p and q are true? Examine by

a) simplifying the expression by using known tautologies (syntactic method)

b) using a truth table (semantic method).

4. Danish philosopher Søren Kierkegaard presented the following reasoning in the 19th century:

> Marry, and you will regret it; don't marry, you will also regret it;
>
> therefore, marry or don't marry, you will regret it either way.

Formalize this reasoning by the means of proportional logic and examine whether it is correct or not.

5. Formalize the following reasonings by the means of proportional logic and find out whether they are correct or not. Use either a truth table (semantic method) or simplification by using known tautologies (syntactic method).

> a) If there is fuel in the car, I go to the store. If I go to the store, I buy cookies. There is fuel in the car. Therefore, I buy cookies.
>
> b) If I study hard or get rich, I get good grades. I get good grades. Therefore, if I don't study hard, I get rich.

6. Are the following expressions true or false?

> a) $\forall x \in \mathbb{Z}_+ : (1+x)^2 > 1 + 2x$ b) $\forall x \in \mathbb{R} : (1+x)^2 > 1 + 2x$
>
> c) $\forall x \in \mathbb{Z} : \exists y \in \mathbb{Z} : x + y = 2x - y$

Answers/hints to selected problems:

1c) There's only one 0 in the final column

3) No, it isn't

4) Correct

5a) Correct *5b) Not correct*