Mathematics A
Assignment two (week 38)

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1. Let U be a set and let $\mathcal{F}_1$ and $\mathcal{F}_2$ be nonempty families of subsets of U such that $\mathcal{F}_1 \subseteq \mathcal{F}_2$. Show (=prove) that the following inclusions hold:

(a) $\bigcup \mathcal{F}_1 \subseteq \bigcup \mathcal{F}_2$
(b) $\bigcap \mathcal{F}_2 \subseteq \bigcap \mathcal{F}_1$

2. Let U be a set and let $\emptyset \neq \mathcal{F} \subseteq \wp(U)$ be a nonempty family of subsets of U. Prove the following equalities:

(a) $(\bigcap \mathcal{F})^c = \bigcup \{A^c \mid A \in \mathcal{F}\}$
(b) $(\bigcup \mathcal{F})^c = \bigcap \{A^c \mid A \in \mathcal{F}\}$

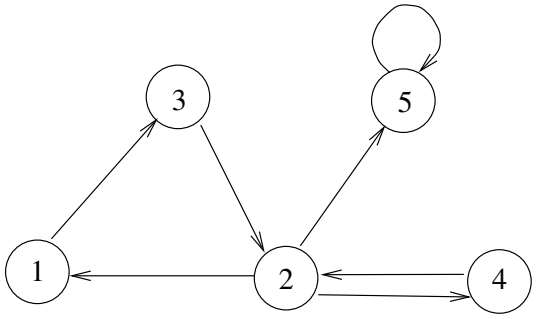
Recall that the complement of any $X \subseteq U$ is defined by $X^c = U \setminus X$.

3. The courses taken by John, Mary, Paul, and Sally are listed below:

John: MATH 211, CSIT 121, MATH 220
Mary: MATH 230, CSIT 121, MATH 212
Paul: CSIT 120, MATH 230, MATH 220
Sally: MATH 211, CSIT 120

Give a graphical representation of the relation R defined as $a\,R\,b$ if student a is taking course b.

4. Write the set of ordered pairs for the relation represented by the following directed graph:



5. Let R be a binary relation on the set $\wp(\{a, b\})$ defined so that $(A, B) \in R$ holds if $A \cap B = \emptyset$. Write out the relation R.

6. Let A, B, C be sets. Prove the following equalities:

(a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$