Mathematics A Assignment three (week 39)

Group 1 (Thu 30/9, 16–18), Group 2 (Fri 1/10, 8–10), Group 3 (Fri 1/10, 10–12)

1. Let $X = \{4, 5, 6\}$, $Y = \{a, b, c\}$ and $Z = l, m, n$. Consider the relations R from X to Y and S from Y to Z defined by:

$$R = \{(4, a), (4, b), (5, c), (6, a), (6, c)\}$$
$$S = \{(a, l), (a, n), (b, l), (b, m), (c, l), (c, m), (c, n)\}.$$

Find the following compositions of relations:

(a) $R \circ S$
(b) $R \circ R^{-1}$

2. Let us denote $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$, that is, $\mathbb{R}^+$ is the set of all positive real numbers.

(a) Let us define a binary relation T on $\mathbb{R}^+$ so that T consists of the pairs (x, x^2), where $x \in \mathbb{R}^+$. What is the inverse relation of T?
(b) Let us define a binary relation S on $\mathbb{R}$ so that S consists of the pairs $(x, 3x - 1)$, where $x \in \mathbb{R}$. What is the inverse relation of S?

3. Assume that A is the set of all human beings. Let the *parent relation* P be a binary relation on A defined by $(a, b) \in P \iff a$ is a parent of b. How do you express using relation P and the operations $\circ$ and $^{-1}$:

(a) the relation G defined so that $(a, b) \in G \iff a$ is a grandparent of b?
(b) the relation S defined so that $(a, b) \in S \iff a$ is a sibling (= sister or brother) of b?
(c) the relation C defined so that $(a, b) \in C \iff a$ is a cousin of b?

4. Let A and B be sets. Suppose that R and S are relations from A to B, that is, $R, S \subseteq A \times B$. Prove that

$$(R \cup S)^{-1} = R^{-1} \cup S^{-1}.$$

5. In lectures it was shown that $P \Rightarrow Q$ is logically equivalent to $\neg P \vee Q$. In other words, the operation $\Rightarrow$ can be defined by using $\vee$ and $\neg$.

(a) Define $\vee$ using $\neg$ and $\Rightarrow$
(b) Define $\wedge$ using $\neg$ and $\vee$.

Prove the correctness of your answer by using truth tables.

6. Define the operation $\vee$ using $\Rightarrow$ only. Prove the correctness by using truth tables.