Mathematics A Assignment four (week 40)

Group 1 (Wed 6/10, 10–12), Group 2 (Fri 8/10, 8–10), Group 3 (Fri 8/10, 12–14)

1. Let P, Q, R and S be logical propositions.

(a) Suppose P is FALSE, Q is FALSE, S is TRUE. Find the truth value of $(S \vee P) \wedge (Q \wedge \neg S)$

(b) Suppose P is TRUE, Q is TRUE, R is FALSE, S is FALSE. Find the truth value of $(Q \vee P) \wedge (\neg R \vee \neg S)$

2. Let P, Q, R and S be logical propositions.

(a) Suppose P is FALSE, S is FALSE, R is TRUE. Find the truth value of $\neg\big((S \wedge P) \vee \neg R\big)$

(b) Suppose P is TRUE, Q is FALSE, R is TRUE. Find the truth value of $P \Rightarrow (Q \Leftrightarrow R)$

3.

(a) How do you write the decimal number 79 in binary form? Use the method described in lecture notes.

(b) What decimal number corresponds the binary number 111 1110 0101?

4. Let $U = \{a, b, c, d, e, f\}$. What are the binary vectors corresponding to the sets:

(a) $\emptyset$ and U,

(b) $A = \{a, c, d, f\}$ and $B = \{a, b, e, f\}$,

(c) $A \cup B$ and $A \cap B$,

(d) A^c and $B \setminus A$.

5. Prove (using direct proof) that for all integers a, b and c, if $a|b$ and $b|c$ then $a|c$. Here $x|y$ (read "x divides y") means that y is a multiple of x (so x will divide into y without remainder).

(Exercise 6 in the next page . . .)

6. Decide which of the following are valid proofs of the following statement:

If ab is an even number, then a or b is even.

Justify your opinion in each case.

(a) Suppose a and b are odd. That is, $a = 2k + 1$ and $b = 2m + 1$ for some integers k and m. Then

$$ab = (2k + 1)(2m + 1) = 4km + 2k + 2m + 1 = 2(2km + k + m) + 1.$$

Therefore ab is odd.

(b) Assume that a or b is even – say it is a (the case where b is even will be identical). That is, $a = 2k$ for some integer k. Then

$$ab = (2k)b = 2(kb).$$

Thus ab is even.

(c) Suppose that ab is even but a and b are both odd. Therefore, $ab = 2n$, $a = 2k+1$ and $b = 2j + 1$ for some integers n, k, and j. Then $2n = (2k + 1)(2j + 1) = 4kj + 2k + 2j + 1$. From this we get

$$n = 2kj + k + j + \frac{1}{2}.$$

But since $2kj + k + j$ is an integer, this says that the integer n is equal to a non-integer, which is impossible.

(d) Let ab be an even number, say $ab = 2n$, and a be an odd number, say $a = 2k+1$. We have $2n = ab = (2k + 1)b = 2kb + b$ and

$$b = 2n - 2kb = 2(n - kb).$$

Therefore b must be even.