

1. Let  $U$  be a universal set and  $A, B \subseteq U$ . Prove the following logical equivalence:

$$A \subseteq B \text{ if and only if } A \setminus B = \emptyset$$

2. Let  $A = \{1, 2, 3, 4\}$  and  $B = \{a, b, c, d\}$ . Which of the following relations are (i) functions, (ii) injections, (iii) surjections, (iv) bijections?

(a)  $R_1 = \{(1, a), (2, c), (3, b), (4, d)\}$

(b)  $R_2 = \{(1, a), (2, b), (3, c), (4, c)\}$

(c)  $R_3 = \{(1, a), (2, b), (3, c), (3, d)\}$

3. Solve  $x$  in the following equations:

(a)  $2^{5x-2} = 16$

(b)  $5 \log_7 x = 10$

(c)  $\log_2(3x - 7) = 5$

(d)  $\log_4 x + \log_4(x - 6) = 2$

4. Prove: There are no integers  $x$  and  $y$  such that  $x^2 = 4y + 2$ . Hint: prove by contradiction

5. Prove: For all integers  $n$ , if  $5n$  is odd, then  $n$  is odd. Clearly state the style of proof you are using.

6. Prove:  $\log_{10}(7)$  is irrational. You probably need the fact that: *if  $n$  is odd, then  $n^k$  is odd for any  $k \in \mathbb{N}$ .*