Mathematics A Assignment 7 (week 44)

Group 1 (Tue 2/11, 12–14), Group 2 (We 3/11, 13–15), Group 3 (Fri 5/11, 12–14)

1. Find the values
$$\binom{70}{5} \qquad \text{and} \qquad \binom{121}{115}.$$
Explain how you simplified the formulas.

2. Expand
$$(1+x)^7.$$

3. Let us denote by $F(\mathbb{N})$ the family of all *finite* subsets of $\mathbb{N}$, that is,
$$F(\mathbb{N}) = \{X \subseteq \mathbb{N} \mid X \text{ is finite}\}.$$
This means that $\{1, 2, 3, \ldots, 1000\} \in F(\mathbb{N})$, but $\{0, 2, 4, 6, 8, \ldots\}$ (even numbers) does not belong to the family $F(\mathbb{N})$.
Show that $F(\mathbb{N})$ is enumerable by describing a suitable enumeration and arguing that each member of $F(\mathbb{N})$ sooner or later appears in your enumeration.

4. Show that the map $f \colon \mathbb{Z} \to \mathbb{N}$ is a bijection:
$$f(n) = \begin{cases} 2n & \text{if } n \geq 0 \\ -2n - 1 & \text{if } n < 0 \end{cases}$$

5. Prove that the cardinality of the interval
$$(0, 1) = \{x \in \mathbb{R} \mid 0 < x < 1\}$$
is the same as the cardinality of $\mathbb{R}$. **Hint:** Use the function f of Example 21.

6. Does $|\mathbb{C}| = |\mathbb{R}|$ hold? Justify your opinion!
Knowing the **Schröder–Bernstein theorem** is probably useful: if there exist injective functions $f \colon A \to B$ and $g \colon B \to A$ between the sets A and B, they have the same cardinality.