**Mathematics A** Assignment 8 (week 45)

Group 1 (Thu 11/11, 10–12), Group 2 (Thu 11/11, 12–14), Group 3 (Fri 12/11, 8–10)

**1.** Prove the *Pascal's rule.* It states that for positive natural numbers $n$ and $k$,

$$\binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n}{k}.$$

**2.** Prove that for any positive integer $n$,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1).$$

**3.** Prove that for any integer $n \geq 2$,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{(n-1) \cdot n} = \frac{n-1}{n}$$

**4.** Prove that for any integer $n \geq 2$,

$$\frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < 1$$

**5.** Prove that Bernoulli's inequality

$$(1+x)^n \geq 1 + nx$$

holds for every real number $x \geq -1$ and every positive integer $n$.

**6.** The **Fibonacci numbers** $F_n$, $n \geq 0$, are such that each number is the sum of the two preceding ones, starting from 0 and 1, that is, $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. The first Fibonacci numbers $F_n$ are:

| $F_0$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | $F_8$ | $F_9$ | $F_{10}$ | $F_{11}$ | $F_{12}$ | $F_{13}$ | $F_{14}$ | $F_{15}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 | 233 | 377 | 610 |

Prove that for all $n \in \mathbb{N}$,

$$F_0 + F_1 + F_2 + \cdots + F_n = F_{n+2} - 1.$$