Mathematics A Assignment 9 (week 46)

Group 1 (Wed 17/11, 17–19), Group 2 (Thu 18/11, 12–14), Group 3 (Fri 19/11, 8–10)

1. For the given a and n, show that $a|n$ by finding an integer k with $n = ak$.

(a) $4|20$
(b) $5|-25$
(c) $-3|9$
(d) $-9|-27$
(e) $1|23$
(f) $-1|17$
(g) $-5|0$
(h) $75|75$

2. Prove, directly from the definition of '|', that for any integer $x \neq 0$,

(a) $x|0$
(b) $1|x$
(c) $x|x$

3. (a) Find all integers n such that $n|(2n+3)$. Are you sure that these are all such integers?

(b) If n is an integer, let $\langle n\rangle$ be the set of all multiples of n. This means that $a \in \langle n\rangle$ whenever there is an integer k such that $a = kn$. Prove that $m|n$ if and only if $\langle n\rangle \subseteq \langle m\rangle$.

4. Determine the greatest common divisor of 2016 and 323, and find integers x and y with $2016x + 323y = \gcd(2016, 323)$.

5. A number l is called a common multiple of m and n if both m and n divide l. The smallest such l is called the **least common multiple** of m and n and is denoted by $\text{lcm}(m, n)$.

(a) Find $\text{lcm}(8, 12)$, $\text{lcm}(20, 30)$, $\text{lcm}(51, 68)$, $\text{lcm}(23, 18)$
(b) Compare the value of $\text{lcm}(m, n)$ with the values of m, n and $\gcd(m, n)$. In what way are they related?
(c) Compute $\text{lcm}(301337, 307829)$ using the formula you found in (b).

6. Prove the formula you found in 5(b).