Task 1

Resistances in the system of resistors	$R_1, R_2, R_3, R_4, R_5, R_6$	[Ω]
Resistance of system of resistors R_1 and R_2	R_{12}	[Ω]
Resistance of system of resistors R_4 and R_5	R_{45}	[Ω]
Resistance of system of resistors R_1, R_2 and R_3	R_{123}	[Ω]
Resistance of system of resistors R_1, R_2, R_3,R_4 and R_5	R_{12345}	[Ω]
Resistance of the system of resistors	R	[Ω]
Voltage over the system of resistors	U	[V]
Current through the system of resistors	I	[A]

Formulas:

$$R_{12} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$R_{123} = R_{12} + R_3$$

$$R_{45} = R_4 + R_5$$

$$R_{12345} = \frac{1}{\frac{1}{R_{123}}+\frac{1}{R_{45}}}$$

$$R = R_{12345} + R_6$$

$$U = RI$$

Solution:

Resistance of the system:

$$R = R_{12345} + R_6$$

substitute expression

$$R = \frac{1}{\frac{1}{R_{123}}+\frac{1}{R_{45}}} + R_6$$

substitute expressions

$$R = \frac{1}{\frac{1}{R_{12}+R_3}+\frac{1}{R_4+R_5}} + R_6$$

substitute expression

$$\boldsymbol{R = \frac{1}{\frac{1}{\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}+R_3}+\frac{1}{R_4+R_5}} + R_6}$$

Substitute initial values

Current flowing through the system:

$U = RI$

solve for I

$I = \frac{U}{R}$

substitute expression

$$\boldsymbol{I = \frac{U}{\frac{1}{\frac{1}{\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}+R_3}+\frac{1}{R_4+R_5}}+R_6}}$$

Substitute initial values

Task 2

Resistances in the system of resistors	$R_1, R_2, R_3, R_4, R_5, R_6, R_7$	[Ω]
Resistance of system of resistors R_1, R_2 and R_3	R_{123}	[Ω]
Resistance of system of resistors R_4 and R_5	R_{45}	[Ω]
Resistance of system of resistors R_1,R_2,R_3 and R_6	R_{1236}	[Ω]
Resistance of system of resistors R_4,R_5 and R_7	R_{457}	[Ω]
Resistance of the system of resistors	R	[Ω]
Voltage over the system of resistors	U	[V]
Voltage over the system of resistors R_1, R_2 and R_3	U_{123}	[V]
Voltage over the system of resistors R_1,R_2,R_3 and R_6	U_{1236}	[V]
Voltage over resistor R_1	U_1	[V]
Voltage over resistor R_6	U_6	[V]
Voltage over resistor R_7	U_7	[V]
Current through the system of resistors	I	[A]
Current through the system of resistors R_1, R_2 and R_3	I_{123}	[A]
Current through the system of resistors R_4, R_5 and R_7	I_{457}	[A]
Current through resistor R_1	I_1	[A]
Current through resistor R_6	I_6	[A]
Current through resistor R_7	I_7	[A]
Power consumption of resistor R_7	P_7	[W]

Formulas:

$$U_1 = R_1 I_1$$

$$U_{123} = U_1$$

$$U_{123} = R_{123} I_{123}$$

$$I_6 = I_{123} = I_{1236}$$

$$U_{1236} = R_{1236} I_{1236}$$

$$U = U_{1236}$$

$$R_{123} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$R_{1236} = R_{123} + R_6$$

$$R_{45} = \frac{1}{\frac{1}{R_4}+\frac{1}{R_5}}$$

$$R_{457} = R_{45} + R_7$$

$$P_7 = U_7 I_7 = R_7 I_7^2$$

$$U = R_{457} I_{457}$$

$$I_7 = I_{457}$$

Solution:

Voltage over the system of resistors:

$U_{123} = R_{123}I_{123}$

solve for I_{123}

$I_{123} = \frac{U_{123}}{R_{123}}$

$U = U_{1236}$

substitute expression

$U = R_{1236}I_{1236}$

substitute expression

$U = R_{1236}I_{123}$

substitute expression

$U = \frac{R_{1236}U_{123}}{R_{123}}$

substitute expression

$U = \frac{(R_{123}+R_6)U_{123}}{R_{123}}$

use algebra

$U = \left(1 + \frac{R_6}{R_{123}}\right)U_{123}$

substitute expressions

$U = \left(1 + \frac{R_6}{R_{123}}\right)U_{123} = \left(1 + \frac{R_6}{R_{123}}\right)U_1 = \left(1 + \frac{R_6}{R_{123}}\right)R_1I_1$

substitute expression

$\boldsymbol{U = \left[1 + R_6\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\right]R_1I_1}$

substitute initial values

Power consumption of resistor R_7

$U = R_{457}I_{457}$

solve for I_{457}

$I_{457} = \frac{U}{R_{457}}$

$P_7 = R_7I_7^2$

substitute expressions

$$P_7 = R_7 I_{457}^2 = R_7\left(\frac{U}{R_{457}}\right)^2 = R_7\left(\frac{U}{R_{45}+R_7}\right)^2 = R_7\left(\frac{U}{\frac{1}{\frac{1}{R_4}+\frac{1}{R_5}}+R_7}\right)^2$$

substitute expression

$$\boldsymbol{P_7 = R_7\left(\frac{\left[1+R_6\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\right]R_1 I_1}{\frac{1}{\frac{1}{R_4}+\frac{1}{R_5}}+R_7}\right)^2}$$

substitute initial values

Task 3

Resistances in the circuit	R_1, R_2, R_3, R_4, R_5	[Ω]
Electromotive forces of the voltage sources	$\varepsilon_1, \varepsilon_2$	[V]
Currents through the resistors	I_1, I_2, I_3, I_4, I_5	[A]

Formulas:

Kirchoff's II law, left loop, clockwise:

$$\varepsilon_2 - R_2 I_2 - R_3 I_3 - R_5 I_5 = 0$$

Kirchoff's II law, right loop, counterclockwise:

$$\varepsilon_1 - R_1 I_1 - R_3 I_3 - R_4 I_4 = 0$$

Kirchoff's I law, where currents I_2 and I_1 merge into I_3:

$$I_1 + I_2 = I_3$$

$$I_2 = I_5$$

$$I_1 = I_4$$

Solution:

Substitute expressions and rearrange

$$\begin{cases} & (R_2 + R_5) I_2 & + R_3 I_3 & = \varepsilon_2 \\ (R_1 + R_4) I_1 & & + R_3 I_3 & = \varepsilon_1 \\ I_1 + & I_2 & - I_3 & = 0 \end{cases}$$

Solve system of equations:

Arrange coefficients of I_1, I_2 and I_3 as a matrix and ε_2, ε_1 and 0 as a vector

$$\begin{pmatrix} 0 & R_2 + R_5 & R_3 \\ R_1 + R_4 & 0 & R_3 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_2 \\ \varepsilon_1 \\ 0 \end{pmatrix}$$

Apply Cramer's rule:

$$I_1 = \frac{\begin{vmatrix} \varepsilon_2 & R_2+R_5 & R_3 \\ \varepsilon_1 & 0 & R_3 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 0 & R_2+R_5 & R_3 \\ R_1+R_4 & 0 & R_3 \\ 1 & 1 & -1 \end{vmatrix}}$$

$$I_2 = \frac{\begin{vmatrix} 0 & \varepsilon_2 & R_3 \\ R_1+R_4 & \varepsilon_1 & R_3 \\ 1 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 0 & R_2+R_5 & R_3 \\ R_1+R_4 & 0 & R_3 \\ 1 & 1 & -1 \end{vmatrix}}$$

$$I_3 = \frac{\begin{vmatrix} 0 & R_2+R_5 & \varepsilon_2 \\ R_1+R_4 & 0 & \varepsilon_1 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 0 & R_2+R_5 & R_3 \\ R_1+R_4 & 0 & R_3 \\ 1 & 1 & -1 \end{vmatrix}}$$

Expand the determinants as follows

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - afh - bdi - ceg$$

$$I_1 = \frac{R_3\varepsilon_1 - R_3\varepsilon_2 + (R_2+R_5)\varepsilon_1}{R_3(R_2+R_5)+R_3(R_1+R_4)+(R_2+R_5)(R_1+R_4)}$$

$$I_2 = \frac{R_3\varepsilon_2 + (R_1+R_4)\varepsilon_2 - R_3\varepsilon_1}{R_3(R_2+R_5)+R_3(R_1+R_4)+(R_2+R_5)(R_1+R_4)}$$

$$I_3 = \frac{(R_2+R_5)\varepsilon_1 + (R_1+R_4)\varepsilon_2}{R_3(R_2+R_5)+R_3(R_1+R_4)+(R_2+R_5)(R_1+R_4)}$$

Use algebra

$$\boldsymbol{I_1} = \frac{(\boldsymbol{R_2}+\boldsymbol{R_3}+\boldsymbol{R_5})\boldsymbol{\varepsilon_1} - \boldsymbol{R_3}\boldsymbol{\varepsilon_2}}{(\boldsymbol{R_1}+\boldsymbol{R_2}+\boldsymbol{R_4}+\boldsymbol{R_5})\boldsymbol{R_3}+(\boldsymbol{R_2}+\boldsymbol{R_5})(\boldsymbol{R_1}+\boldsymbol{R_4})}$$

$$\boldsymbol{I_2} = \frac{(\boldsymbol{R_1}+\boldsymbol{R_3}+\boldsymbol{R_4})\boldsymbol{\varepsilon_2} - \boldsymbol{R_3}\boldsymbol{\varepsilon_1}}{(\boldsymbol{R_1}+\boldsymbol{R_2}+\boldsymbol{R_4}+\boldsymbol{R_5})\boldsymbol{R_3}+(\boldsymbol{R_2}+\boldsymbol{R_5})(\boldsymbol{R_1}+\boldsymbol{R_4})}$$

$$\boldsymbol{I_3} = \frac{(\boldsymbol{R_2}+\boldsymbol{R_5})\boldsymbol{\varepsilon_1} + (\boldsymbol{R_1}+\boldsymbol{R_4})\boldsymbol{\varepsilon_2}}{(\boldsymbol{R_1}+\boldsymbol{R_2}+\boldsymbol{R_4}+\boldsymbol{R_5})\boldsymbol{R_3}+(\boldsymbol{R_2}+\boldsymbol{R_5})(\boldsymbol{R_1}+\boldsymbol{R_4})}$$

$$I_1 = I_4$$

$$I_2 = I_5$$

substitute expressions

$$\boldsymbol{I_4} = \frac{(\boldsymbol{R_2}+\boldsymbol{R_3}+\boldsymbol{R_5})\boldsymbol{\varepsilon_1} - \boldsymbol{R_3}\boldsymbol{\varepsilon_2}}{(\boldsymbol{R_1}+\boldsymbol{R_2}+\boldsymbol{R_4}+\boldsymbol{R_5})\boldsymbol{R_3}+(\boldsymbol{R_2}+\boldsymbol{R_5})(\boldsymbol{R_1}+\boldsymbol{R_4})}$$

$$\boldsymbol{I_5} = \frac{(\boldsymbol{R_1}+\boldsymbol{R_3}+\boldsymbol{R_4})\boldsymbol{\varepsilon_2} - \boldsymbol{R_3}\boldsymbol{\varepsilon_1}}{(\boldsymbol{R_1}+\boldsymbol{R_2}+\boldsymbol{R_4}+\boldsymbol{R_5})\boldsymbol{R_3}+(\boldsymbol{R_2}+\boldsymbol{R_5})(\boldsymbol{R_1}+\boldsymbol{R_4})}$$

substitute initial values

Task 4

Electromotive force of the voltage source	ε	[V]
Internal resistance of the voltage source	r	[Ω]
External resistances in the circuit	R_1, R_2	[Ω]
Current through the circuit when switch is open	I_{open}	[A]
Current through the circuit when switch is closed	I_{closed}	[A]

Formulas:

When the switch is closed, the voltage drops over the whole circuit equal to zero

$$\varepsilon - rI_{\text{closed}} - \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} I_{\text{closed}} = 0$$

When the switch is open, the voltage drops over the whole circuit equal to zero

$$\varepsilon - rI_{\text{open}} - R_1 I_{\text{open}} = 0$$

Solution:

Rearrange the voltage drop equations

$$\begin{cases} \varepsilon - I_{\text{closed}} r = \dfrac{I_{\text{closed}}}{\frac{1}{R_1}+\frac{1}{R_2}} \\ \varepsilon - I_{\text{open}} r = R_1 I_{\text{open}} \end{cases}$$

Arrange left-side coefficients as a matrix and right sides as a vector

$$\begin{pmatrix} 1 & -I_{\text{closed}} \\ 1 & -I_{\text{open}} \end{pmatrix}$$

$$\begin{pmatrix} \dfrac{I_{\text{closed}}}{\frac{1}{R_1}+\frac{1}{R_2}} \\ R_1 I_{\text{open}} \end{pmatrix}$$

Use Cramer's rule

$$\varepsilon = \frac{\begin{vmatrix} \frac{I_{\text{closed}}}{\frac{1}{R_1}+\frac{1}{R_2}} & -I_{\text{closed}} \\ R_1 I_{\text{open}} & -I_{\text{open}} \end{vmatrix}}{\begin{vmatrix} 1 & -I_{\text{closed}} \\ 1 & -I_{\text{open}} \end{vmatrix}} = \frac{R_1 I_{\text{open}} I_{\text{closed}} - \frac{I_{\text{open}} I_{\text{closed}}}{\frac{1}{R_1}+\frac{1}{R_2}}}{I_{\text{closed}} - I_{\text{open}}} = \frac{I_{\text{open}} I_{\text{closed}} \left(R_1 - \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} \right)}{I_{\text{closed}} - I_{\text{open}}}$$

$$r = \frac{\begin{vmatrix} 1 & \frac{I_{\text{closed}}}{\frac{1}{R_1}+\frac{1}{R_2}} \\ 1 & R_1 I_{\text{open}} \end{vmatrix}}{\begin{vmatrix} 1 & -I_{\text{closed}} \\ 1 & -I_{\text{open}} \end{vmatrix}} = \frac{R_1 I_{\text{open}} - \frac{I_{\text{closed}}}{\frac{1}{R_1}+\frac{1}{R_2}}}{I_{\text{closed}} - I_{\text{open}}}$$

use algebra

$$\boldsymbol{\varepsilon} = \frac{R_1 - \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}}{\frac{1}{I_{\text{open}}} - \frac{1}{I_{\text{closed}}}}$$

$$\boldsymbol{r} = \frac{R_1 I_{\text{open}} - \frac{I_{\text{closed}}}{\frac{1}{R_1}+\frac{1}{R_2}}}{I_{\text{closed}} - I_{\text{open}}}$$

substitute initial values

Task 5

Resistances in the circuit	R_1, R_2, R_3	[Ω]
Internal resistances	r_1,r_2	[Ω]
Electromotive forces of the voltage sources	ε_1,ε_2	[V]
Currents through the resistors	I_1,I_2,I_3	[A]
Power consumption in resistor R_3	P_3	[W]

Formulas:

Kirchoff's II law, left loop, clockwise:

$$\varepsilon_1 - r_1 I_1 - R_1 I_1 - R_3 I_3 = 0$$

Kirchoff's II law, right loop, counterclockwise:

$$\varepsilon_2 - r_2 I_2 - R_2 I_2 - R_3 I_3 = 0$$

Kirchoff's I law, where currents I_2 and I_1 merge into I_3:

$$I_1 + I_2 = I_3$$

Power consumption in resistor R_3

$$P_3 = R_3 I_3^2$$

Solution:

Substitute expressions and rearrange

$$\begin{cases} (r_1 + R_1) I_1 & & + R_3 I_3 & = \varepsilon_1 \\ & (r_2 + R_2) I_2 & + R_3 I_3 & = \varepsilon_2 \\ I_1 + & I_2 & - I_3 & = 0 \end{cases}$$

Solve system of equations:

Arrange coefficients of I_1, I_2 and I_3 as a matrix and ε_2, ε_1 and 0 as a vector

$$\begin{pmatrix} r_1 + R_1 & 0 & R_3 \\ 0 & r_2 + R_2 & R_3 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ 0 \end{pmatrix}$$

Apply Cramer's rule:

$$I_1 = \frac{\begin{vmatrix} \varepsilon_1 & 0 & R_3 \\ \varepsilon_2 & r_2+R_2 & R_3 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} r_1+R_1 & 0 & R_3 \\ 0 & r_2+R_2 & R_3 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{-\varepsilon_1(r_2+R_2)+\varepsilon_2 R_3-\varepsilon_1 R_3}{-(r_1+R_1)(r_2+R_2)-(r_1+R_1)R_3-R_3(r_2+R_2)}$$

$$I_2 = \frac{\begin{vmatrix} r_1+R_1 & \varepsilon_1 & R_3 \\ 0 & \varepsilon_2 & R_3 \\ 1 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} r_1+R_1 & 0 & R_3 \\ 0 & r_2+R_2 & R_3 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{-(r_1+R_1)\varepsilon_2+R_3\varepsilon_1-R_3\varepsilon_2}{-(r_1+R_1)(r_2+R_2)-(r_1+R_1)R_3-R_3(r_2+R_2)}$$

$$I_3 = \frac{\begin{vmatrix} r_1+R_1 & 0 & \varepsilon_1 \\ 0 & r_2+R_2 & \varepsilon_2 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} r_1+R_1 & 0 & R_3 \\ 0 & r_2+R_2 & R_3 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{-(r_1+R_1)\varepsilon_2-(r_2+R_2)\varepsilon_1}{-(r_1+R_1)(r_2+R_2)-(r_1+R_1)R_3-R_3(r_2+R_2)}$$

use algebra

$$\boldsymbol{I_1 = \frac{(r_2+R_2+R_3)\varepsilon_1-R_3\varepsilon_2}{R_3(r_1+R_1+r_2+R_2)+(r_1+R_1)(r_2+R_2)}}$$

$$\boldsymbol{I_2 = \frac{\varepsilon_2(r_1+R_1+R_3)-R_3\varepsilon_1}{R_3(r_1+R_1+r_2+R_2)+(r_1+R_1)(r_2+R_2)}}$$

$$\boldsymbol{I_3 = \frac{(r_1+R_1)\varepsilon_2+(r_2+R_2)\varepsilon_1}{R_3(r_1+R_1+r_2+R_2)+(r_1+R_1)(r_2+R_2)}}$$

$$\boldsymbol{P_3 = R_3\left(\frac{(r_1+R_1)\varepsilon_2+(r_2+R_2)\varepsilon_1}{R_3(r_1+R_1+r_2+R_2)+(r_1+R_1)(r_2+R_2)}\right)^2}$$

substitute initial values