Task 1

Thermal conductivity	k	$[\frac{\mathrm{W}}{\mathrm{mk}}]$
Inside temperature	T_{in}	$[^{\circ}\mathrm{C}]$
Outside temperature	T_{out}	$[^{\circ}\mathrm{C}]$
Wall width	w	$[\mathrm{m}]$
Wall height	h	$[\mathrm{m}]$
Wall surface area	A	$[\mathrm{m}^2]$
Wall thickness	d	$[\mathrm{m}]$
Heat flow through the wall	Φ	$[W]$

Equations:

$$\Phi = \frac{k}{d}A(T_{\mathrm{out}} - T_{\mathrm{in}})$$

$$A = wh$$

Solution:

$$\Phi = \frac{kwh}{d}(T_{\mathrm{out}} - T_{\mathrm{in}})$$

$$= \frac{\left(0.8\frac{\mathrm{W}}{\mathrm{mk}}\right)(8\ \mathrm{m})(3\ \mathrm{m})}{(0.15\ \mathrm{m})}\big((35\ ^{\circ}\mathrm{C}) - (22\ ^{\circ}\mathrm{C})\big) = 1{,}664\ \mathrm{W} = 1.664\ \mathrm{kW}$$

Task 2

Heat released by the battery	Q	[J]
Temperature of the battery	T	[K]
Entropy generated as the heat is released	S	[J/K]

Equations:

$$S = \int_{Q_0}^{Q} \frac{dQ}{T}$$

Solution:

$$S = \int_{Q_0}^{Q} \frac{Q_0 - 3\ln Q + \frac{22Q}{Q^2+4}}{K_0} dQ = \frac{1}{K_0}\left[Q_0 \int_{Q_0}^{Q} dQ - 3\int_{Q_0}^{Q} \ln Q \, dQ + 22\int_{Q_0}^{Q} \frac{Q}{Q^2+4} dQ \right]$$

$$= \frac{1}{K_0}\left[Q_0(Q - Q_0) - 3(Q\ln Q - Q)\Big|_{Q_0}^{Q} + 22\frac{1}{2}[\ln(Q^2+4)]\Big|_{Q_0}^{Q}\right]$$

$$= \frac{1}{K_0}\left[Q_0 Q - Q_0^2 - 3(Q\ln Q - Q) + 3(Q_0 \ln Q_0 - Q_0) + 11\ln\frac{Q^2+4}{Q_0^2+4}\right]$$

Task 3

Thermal conductivity of brick	k_b	$[\frac{\mathrm{W}}{\mathrm{mK}}]$
Thermal conductivity of styrofoam	k_s	$[\frac{\mathrm{W}}{\mathrm{mK}}]$
Inside temperature	T_in	[°C]
Outside temperature	T_out	[°C]
Temperature between brick and styrofoam	T_w	[°C]
Brick wall thickness	d_b	[m]
Styrofoam thickness	d_s	[m]
Heat flow through the wall per unit area	φ	[W/A]

Equations:

$$\varphi = \frac{k_\mathrm{b}}{d_\mathrm{b}}(T_\mathrm{w} - T_\mathrm{out})$$

$$\varphi = \frac{k_\mathrm{s}}{d_\mathrm{s}}(T_\mathrm{in} - T_\mathrm{w})$$

Solution:

$$\begin{cases} \frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{w} - \varphi = \frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{out} \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{w} + \varphi = \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{in} \end{cases}$$

$$T_\mathrm{w} = \frac{\begin{bmatrix} \frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{out} & -1 \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{in} & 1 \end{bmatrix}}{\begin{bmatrix} \frac{k_\mathrm{b}}{d_\mathrm{b}} & -1 \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} & 1 \end{bmatrix}} = \frac{\frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{out} + \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{in}}{\frac{k_\mathrm{b}}{d_\mathrm{b}} + \frac{k_\mathrm{s}}{d_\mathrm{s}}}$$

$$= \frac{\frac{\left(0.8\frac{\mathrm{W}}{\mathrm{mK}}\right)(-10\ ^\circ\mathrm{C})}{(0.15\ \mathrm{m})} + \frac{\left(0.01\frac{\mathrm{W}}{\mathrm{mK}}\right)(21\ ^\circ\mathrm{C})}{(0.10\ \mathrm{m})}}{\frac{\left(0.8\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.15\ \mathrm{m})} + \frac{\left(0.01\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.10\ \mathrm{m})}} = -9.4294478527 \ldots\ ^\circ\mathrm{C}$$

$$\varphi = \frac{\begin{bmatrix} \frac{k_\mathrm{b}}{d_\mathrm{b}} & \frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{out} \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} & \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{in} \end{bmatrix}}{\begin{bmatrix} \frac{k_\mathrm{b}}{d_\mathrm{b}} & -1 \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} & 1 \end{bmatrix}} = \frac{\frac{k_\mathrm{b} k_\mathrm{s}}{d_\mathrm{b} d_\mathrm{s}}(T_\mathrm{in} - T_\mathrm{out})}{\frac{k_\mathrm{b}}{d_\mathrm{b}} + \frac{k_\mathrm{s}}{d_\mathrm{s}}}$$

$$= \frac{\frac{\left(0.8\frac{\mathrm{W}}{\mathrm{mK}}\right)\left(0.01\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.15\ \mathrm{m})(0.10\ \mathrm{m})}\left((21\ ^\circ\mathrm{C}) - (-10\ ^\circ\mathrm{C})\right)}{\frac{\left(0.8\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.15\ \mathrm{m})} + \frac{\left(0.01\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.10\ \mathrm{m})}} = 3.04294478527 \ldots \frac{\mathrm{W}}{\mathrm{m}^2}$$

Solution is the same if T is in kelvin:

$$\begin{cases} \frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{w} - \varphi = \frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{out} \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{w} + \varphi = \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{in} \end{cases}$$

$$T_\mathrm{w} = \frac{\begin{bmatrix} \frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{out} & -1 \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{in} & 1 \end{bmatrix}}{\begin{bmatrix} \frac{k_\mathrm{b}}{d_\mathrm{b}} & -1 \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} & 1 \end{bmatrix}} = \frac{\frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{out} + \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{in}}{\frac{k_\mathrm{b}}{d_\mathrm{b}} + \frac{k_\mathrm{s}}{d_\mathrm{s}}}$$

$$= \frac{\frac{\left(0.8\frac{\mathrm{W}}{\mathrm{mK}}\right)(273.15-10)\mathrm{K}}{(0.15\ \mathrm{m})} + \frac{\left(0.01\frac{\mathrm{W}}{\mathrm{mK}}\right)(273.15+21)\mathrm{K}}{(0.10\ \mathrm{m})}}{\frac{\left(0.8\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.15\ \mathrm{m})} + \frac{\left(0.01\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.10\ \mathrm{m})}} = 263.72055214\ldots\ \mathrm{K} = -9.429447852\ldots\ ℃$$

$$\varphi = \frac{\begin{bmatrix} \frac{k_\mathrm{b}}{d_\mathrm{b}} & \frac{k_\mathrm{b}}{d_\mathrm{b}} T_\mathrm{out} \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} & \frac{k_\mathrm{s}}{d_\mathrm{s}} T_\mathrm{in} \end{bmatrix}}{\begin{bmatrix} \frac{k_\mathrm{b}}{d_\mathrm{b}} & -1 \\ \frac{k_\mathrm{s}}{d_\mathrm{s}} & 1 \end{bmatrix}} = \frac{\frac{k_\mathrm{b} k_\mathrm{s}}{d_\mathrm{b} d_\mathrm{s}} (T_\mathrm{in} - T_\mathrm{out})}{\frac{k_\mathrm{b}}{d_\mathrm{b}} + \frac{k_\mathrm{s}}{d_\mathrm{s}}}$$

$$= \frac{\frac{\left(0.8\frac{\mathrm{W}}{\mathrm{mK}}\right)\left(0.01\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.15\ \mathrm{m})(0.10\ \mathrm{m})}\left((273.15+21)\mathrm{K} - (273.15-10)\mathrm{K}\right)}{\frac{\left(0.8\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.15\ \mathrm{m})} + \frac{\left(0.01\frac{\mathrm{W}}{\mathrm{mK}}\right)}{(0.10\ \mathrm{m})}} = 3.04294478527\ldots\ \frac{\mathrm{W}}{\mathrm{m^2}}$$

Task 4

Work done by the force	W	[J]
Distance travelled by the object	s	[m]
Velocity component directed along x-axis	v	$[\frac{\mathrm{m}}{\mathrm{s}}]$
Friction force	F	[N]
Time	t	[s]

Equations:

$$dW = F \cdot ds$$

Solution:

$$F = \cos(0.1)(7\ \mathrm{N} + 0.2\frac{\mathrm{N}}{\mathrm{s}}\ t)$$

$$dW = F \cdot ds = F \cdot \frac{ds}{dt} dt = F \cdot v\ dt$$

$$W = \cos(0.1) \int_{t=0}^{t} \left(7\ \mathrm{N} + 0.2\frac{\mathrm{N}}{\mathrm{s}}\ t\right)\left(12\frac{\mathrm{m}}{\mathrm{s}} - 3.5\frac{\mathrm{m}}{\mathrm{s}^2} t + 0.13\ t^2 \frac{\mathrm{m}}{\mathrm{s}^3}\right) dt$$

$$W = \cos(0.1) \int_{t=0}^{t} \begin{matrix} \left(84\ \mathrm{W} - 24.5\frac{\mathrm{W}}{\mathrm{s}} t + 0.91\frac{W}{s^2} t^2\right) \\ +(2.4\frac{\mathrm{W}}{\mathrm{s}} t - 0.7\frac{\mathrm{W}}{\mathrm{s}^2} t^2 + 0.026\frac{\mathrm{W}}{\mathrm{s}^3} t^3) dt \end{matrix}$$

$$W = \cos(0.1) \int_{t=0}^{t} (84\ \mathrm{W} - 22.1\frac{\mathrm{W}}{\mathrm{s}} t + 0.21\frac{\mathrm{W}}{\mathrm{s}^2} t^2 + 0.026\frac{\mathrm{W}}{\mathrm{s}^3} t^3) dt$$

$$W = \cos(0.1)\left[84\ \mathrm{W}\ t - 11.05\frac{\mathrm{W}}{\mathrm{s}}\ t^2 + 0.07\frac{\mathrm{W}}{\mathrm{s}^2}\ t^3 + 0.0065\frac{\mathrm{W}}{\mathrm{s}^4}\ t^4\right]$$

$$W(t = 2s) = \cos(0.1)\,[84(2) - 11.05(2)^2 + 0.07(2)^3 + 0.0065(2)^4]\mathrm{J}$$

$$= 123.8421984271641 \ldots \mathrm{J}$$

Task 5

Heat flow, same through all insulation layers	φ	$[\frac{\mathrm{W}}{\mathrm{m}^2}]$
Area of insulation layers	A	$[\mathrm{m}^2]$
Number of insulation layers	N	[-]
Overall thermal resistance of insulation layers	R	$[\frac{\mathrm{K}}{\mathrm{W}}]$
Temperature difference over all insulation layers	ΔT	[K]
Area of insulation layer i	A_i	$[\mathrm{m}^2]$
Thermal resistance of insulation layer i	R_i	$[\frac{\mathrm{K}}{\mathrm{W}}]$
Temperature difference over insulation layer i	ΔT_i	[K]
Thickness of insulation layer i	d_i	[m]
Thermal conductivity of insulation layer i	k_i	$[\frac{\mathrm{W}}{\mathrm{mK}}]$

Proof:

Heat flux is the same through all insulation layers:

$$\varphi = \frac{k_\mathrm{i}}{d_\mathrm{i}} \Delta T_\mathrm{i}$$

solve for ΔT_i

$$\Delta T_\mathrm{i} = \frac{d_\mathrm{i}}{k_\mathrm{i}} \varphi = R_\mathrm{i} A_\mathrm{i} \varphi$$

Temperature difference over all insulation layers is the sum of all temperature differences:

$$\Delta T = \sum_{\mathrm{i}=1}^{\mathrm{N}} \Delta T_\mathrm{i} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{d_\mathrm{i}}{k_\mathrm{i}} \varphi = \sum_{\mathrm{i}=1}^{\mathrm{N}} R_\mathrm{i} A_\mathrm{i} \varphi$$

Overall thermal resistance is related to ΔT and φ as follows:

$$\Delta T = RA\varphi = \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{d_\mathrm{i}}{k_\mathrm{i}} \varphi = \sum_{\mathrm{i}=1}^{\mathrm{N}} R_\mathrm{i} A_\mathrm{i} \varphi$$

$$\Rightarrow \quad R = \frac{1}{A} \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{d_\mathrm{i}}{k_\mathrm{i}} = \sum_{\mathrm{i}=1}^{\mathrm{N}} R_\mathrm{i}$$