BM20A8800 Discrete Models and Methods 3op

Exercise 3 / Week 5

1. Use proof by induction to show that the following formula is correct for all $n \in \mathbb{Z}_+$.

$$1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

2. Use proof by induction to show that the following inequalities are correct.

a) $n^2 < 2^n$ for all $n \geq 5$ b) $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$ for all $n \geq 2$

3. Let's examine the following recursively defined sequence:

$$p_{n+1} = \frac{(p_n)^2}{2}$$

a) If the 1st term of the sequence is p_0, use the recursive formula in order to calculate terms $p_1 \dots p_5$. (First term p_0 will naturally be left as a parameter.)

b) Use the previous result to heuristically define a candidate for closed form formula for the term p_n.

c) Use proof by induction to show that this closed form formula is correct. (If it wasn't, re-evaluate your candidate in section b.)

4. Solve the following recurrence relations.

a) $y_{n+2} - 3y_{n+1} - 10y_n = 0$ b) $9y_{n+2} - 6y_{n+1} + y_n = 0$, initial conditions $y_0 = 2$ & $y_1 = 3$

5. Solve the following nonhomogeneous recurrence relations.

a) $y_{n+2} - 3y_{n+1} + 2y_n = 3^n$ b) $y_{n+2} - 3y_{n+1} + 2y_n = 5$

6. Solve the following nonhomogeneous recurrence relation.

$$y_{n+3} - 6y_{n+2} + 5y_{n+1} + 12y_n = 6n$$

Answers / hints for selected problems:

1. Hint: simplify the right side to a 3rd degree polynomial and then see whether the left side can be simplified to same form using the induction hypothesis.

2. Hint: Lecture 5, example 2 might provide a good starting point.

3. Hint: 2 is literally a powerful number.

4. a) - *b)* $y_n = 2\left(\frac{1}{3}\right)^n + 7n\left(\frac{1}{3}\right)^n$

5. a) $y_n = C_1 + C_2 \cdot 2^n + \frac{1}{2} \cdot 3^n$ *b) Hint: you'll need to modify the* y_p.

6. $y_n = c_1(-1)^n + c_2 3^n + c_3 4^n + \frac{1}{2}n + \frac{1}{6}$ *(Hint:* $y_{n,p} = An + B$*)*