LUT University

LUT Computational Engineering 2022-03-23

BM40A0202 Foundations of Computer Science

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Exercise 10 (week 13): Automata and Turing machines.

Tasks (1 p/task)

1. In Moodle, you can find a .txt file that includes 100 most common passwords (according to some company – I assume SplashData; I apologize the obscenities, but it's not my fault that people seem to be fond of such words). Copy-paste this data to "Test string" field in www.regexpal.com and then start working on the following questions:

   a) How many passwords have numbers "123" in them (in this order)? Construct a regular expression that finds these passwords.

   b) How many passwords contain at least one number? Construct a regular expression that finds these passwords.

   c) How many passwords contain at least one vowel? Construct a regular expression that finds these passwords. (Note: y is not considered a vowel here.)

   d) How many passwords contain at least two vowels separated by one character (which can also be a vowel)? Construct a regular expression that finds these passwords.

   Tip: RexEgg has a good list of regular expressions here that you can use as a guide. Link: https://www.rexegg.com/regex-quickstart.html

2. (a) Make the transition table of the automaton visualized in Figure 1. What kind of inputs does the automaton accept?

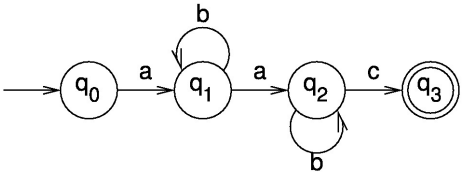

Figure 1: Automaton.

   (b) Draw a state diagram based on the transition table in Table 1. What inputs does the automaton accept?

Table 1: Transition table.

	δ	a	b
$\rightarrow$	q_e	q_f	q_e
	q_f	q_f	q_s
	q_s	q_f	q_o
$\leftarrow$	q_o	q_f	q_e

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3. Let's say we have a Turing machine M that is specified in the following way (starts from right):

$$M = (Q, T, I, \delta, b, q_0, q_f)$$

$$\begin{array}{lll} Q = & \{0, 1, 2, H\} & \delta = \\ T = & \{0, 1, b\} & \\ I = & \{0, 1\} & \\ b = & \{b\} & \\ q_0 = & 0 & \\ q_f = & H & \end{array} \quad \begin{array}{l} 0, \_ \rightarrow 0, \_, L \\ 0, 0 \rightarrow 1, 1, L \\ 0, 1 \rightarrow 1, 0, L \\ 1, 0 \rightarrow 1, 1, L \\ 1, 1 \rightarrow 1, 0, L \\ 1, \_ \rightarrow 2, \_, R \\ 2, 0 \rightarrow 2, 0, R \\ 2, 1 \rightarrow 2, 1, R \\ 2, \_ \rightarrow H, \_, S \end{array}$$

a) Code this into Morphett simulator and run it using the following inputs. How many steps does the machine take until it halts? Add at least one screenshot of a completed run.

i) 1101 ii) 010101 iii) 1001010001011

b) What does the machine do? (If you can't see it based on the previous 3 runs, perform a couple more.)

4. A Turing machine can have multiple tapes instead of just one. Below we have defined a three-tape Turing machine as (machine starts from left side of the input on all tapes):
$M = (Q, T, I, \delta, b, q_s, q_f)$, where

$$\begin{array}{rcl} Q = & \{q_0, q_1, q_2, q_3\} & \text{(set of states)}, \\ T = & \{0, 1, b\} & \text{(set of tape symbols)}, \\ I = & \{0, 1\} & \text{(set of input symbols)}, \\ b = & \{b\} & \text{(empty symbol)}, \\ q_s = & q_0 & \text{(initial state) and} \\ q_f = & q_3 & \text{(final state)} \end{array}$$

The state transitions in δ machine M are $\delta : (Q_s \times T)^3 \rightarrow (Q_n \times T \times \{\mathrm{L}, \mathrm{S}, \mathrm{R}\})^3$:

State Q_s	State Q_n	Tape 1	Tape 2	Tape 3
q_0	q_0	1,1,R	0,0,R	b,b,R
q_0	q_0	0,0,R	1,1,R	b,b,R
q_0	q_0	1,1,R	1,1,R	b,b,R
q_0	q_0	0,0,R	0,0,R	b,b,R
q_0	q_1	b,b,L	b,b,L	b,b,L
q_1	q_1	1,1,L	0,0,L	b,1,L
q_1	q_1	0,0,L	1,1,L	b,1,L
q_1	q_1	0,0,L	0,0,L	b,0,L
q_1	q_2	1,1,L	1,1,L	b,0,L
q_1	q_3	b,b,R	b,b,R	b,b,R
q_2	q_2	1,1,L	1,1,L	b,1,L
q_2	q_2	1,1,L	0,0,L	b,0,L
q_2	q_2	0,0,L	1,1,L	b,0,L
q_2	q_1	0,0,L	0,0,L	b,1,L
q_2	q_3	b,b,R	b,b,R	b,1,L

a) Perform a run with inputs, where tape 1 = $b1010b$ and tape 2 = $b0110b$. Tape 3 is initially empty (so, full of b's). What do you get as result in all 3 tapes? Document (at least some of) the steps in order to show how the machine progresses. You'll have to do this yourself – Morphett can't simulate a multi-tape machine. (Don't worry though, there are not that many steps.)

b) What does the machine do? Can you give the states q_0, q_1 and q_2 names that would explain what happens in them?

5. Define a Turing machine that accepts the language $\{a^i b^i | i \geq 0\}$. (i.e., words with the same number of letters a and b in a row).

Test the machine you have specified with different inputs using the Morphett simulator. Include the code and a screenshot of a successful simulation in your submission. Moreover, be prepared in the exercises to demonstrate the operation of your machine using the simulator in the exercise with the inputs requested by the teacher.