1.

(a) Suppose P is FALSE, Q is FALSE, S is TRUE.

$$(S \vee P) \wedge (Q \wedge \neg S) = (\mathbf{T} \vee \mathbf{F}) \wedge (\mathbf{F} \wedge \neg \mathbf{T}) = \mathbf{T} \wedge \mathbf{F} = \mathbf{F}$$

(b) Suppose P is TRUE, Q is TRUE, R is FALSE, S is FALSE.

$$(Q \vee P) \wedge (\neg R \vee \neg S) = (\mathbf{T} \vee \mathbf{T}) \wedge (\neg \mathbf{F} \vee \neg \mathbf{F}) = \mathbf{T} \wedge \mathbf{T} = \mathbf{T}$$

2. Let P, Q, R and S be logical propositions.

(a) Suppose P is FALSE, S is FALSE, R is TRUE.

$$\neg\big((S \wedge P) \vee \neg R\big) = \neg\big((\mathbf{F} \wedge \mathbf{F}) \vee \neg \mathbf{T}\big) = \neg(\mathbf{F} \vee \mathbf{F}) = \neg \mathbf{F} = \mathbf{T}.$$

(b) Suppose P is TRUE, Q is FALSE, R is TRUE.

$$P \Rightarrow (Q \Leftrightarrow R) \;=\; \mathbf{T} \Rightarrow (\mathbf{F} \Leftrightarrow \mathbf{T}) \;=\; \mathbf{T} \Rightarrow \mathbf{F} = \mathbf{F}.$$

3.

(a)

Div. by 2	Quotient	Remainder
79/2	39	1
39/2	19	1
19/2	9	1
9/2	4	1
4/2	2	0
2/2	1	0
1/2	0	1

$B = 100\,1111$

(b)

Binary	1	1	1	1	1	1	0	0	1	0	1
Position	10	9	8	7	6	5	4	3	2	1	0

$$\begin{aligned} D &= 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^2 + 2^0 \\ &= 1024 + 512 + 256 + 64 + 32 + 4 + 1 = 2021 \end{aligned}$$

4. Let $U = \{a, b, c, d, e, f\}$.

(a) $\emptyset = (0,0,0,0,0,0)$ and $U = (1,1,1,1,1,1)$

(b) $A = (1,0,1,1,0,1)$ and $B = (1,1,0,0,1,1)$

(c) $A \cup B = (1 \vee 1, 0 \vee 1, 1 \vee 0, 1 \vee 0, 0 \vee 1, 1 \vee 1) = (1,1,1,1,1,1)$ and
$A \cap B = (1 \wedge 1, 0 \wedge 1, 1 \wedge 0, 1 \wedge 0, 0 \wedge 1, 1 \wedge 1) = (1,0,0,0,0,1)$.

(d) $A^c = (\neg 1, \neg 0, \neg 1, \neg 1, \neg 0, \neg 1) = (0,1,0,0,1,0)$ and
$B \setminus A = (1 \wedge \neg 1, 1 \wedge \neg 0, 0 \wedge \neg 1, 0 \wedge \neg 1, 1 \wedge \neg 0, 1 \wedge \neg 1) = (0,1,0,0,1,0)$

5. Now $x|y$ means that there is an integer k such that $y = kx$. If $a|b$ and $b|c$, then there are integers k_1 and k_2 such that $b = k_1 a$ and $c = k_2 b$. This means that

$$c = k_2 b = k_2 k_1 a.$$

Because $k_1 k_2$ is an integer, $a|c$.

6. Let P = "ab is even". Then $\neg P$ = "ab is **not** even", that is, $\neg P$ = "ab is odd". Let Q = "a **or** b is even". In fact, Q consists of **two** propositions Q_1 = "a is even" and Q_2 = "b is even". Then, $Q = Q_1 \vee Q_2$ and

$$\begin{aligned}\neg Q &= \neg(Q_1 \vee Q_2) = \neg Q_1 \wedge \neg Q_2 = \text{"}a\text{ is }\textbf{not}\text{ even"} \wedge \text{"}b\text{ is }\textbf{not}\text{ even"}\\ &= \text{"}a\text{ is odd"} \wedge \text{"}b\text{ is odd"} = \text{"}a\text{ and }b\text{ are odd"}\end{aligned}$$

Then, the claim

If ab is an even number, then a or b is even

equals the proposition $P \Rightarrow Q$.

(a) This proves $\neg Q \Rightarrow \neg P$. This is logically equivalent to $P \Rightarrow Q$. Therefore, the proof is valid.
(b) This proves $Q \Rightarrow P$. The proof is not valid.
(c) This supposes that P is true. Then it shows that from $\neg Q$ follows the contradiction **F**. Therefore, Q is true. We have that $P \Rightarrow Q$ is true and the proof is valid.
(d) This also supposes that P is true. Then it shows that $\neg Q_1$ implies Q_2, that is, $\neg Q_1 \Rightarrow Q_2$. Now $\neg Q_1 \Rightarrow Q_2$ is equivalent to $Q_1 \vee Q_2$. Thus, $Q = Q_1 \vee Q_2$ is true and we have shown that $P \Rightarrow Q$ is true. The proof is valid.