1. An integer k with $n = ak$:

(a) $20 = 4 \cdot 5$
(b) $-25 = 5 \cdot (-5)$
(c) $9 = -3 \cdot -3$
(d) $-27 = -9 \cdot 3$
(e) $23 = 1 \cdot 23$
(f) $17 = -1 \cdot (-17)$
(g) $0 = -5 \cdot 0$
(h) $75 = 75 \cdot 1$

2. $a|b$ if there is k such that $b = a \cdot k$.

(a) $x|0$, because $0 = x \cdot 0$
(b) $1|x$, because $x = 1 \cdot x$
(c) $x|x$, because $x = x \cdot 1$

3. We have that

$$\frac{2n+3}{n} = 2 + \frac{3}{n}.$$

The result is an integer if and only if $\frac{3}{n}$ is an integer. This happens exactly when $n \in \{-3, -1, 1, 3\}$.

(b) By definition $a \in \langle b \rangle \iff b|a$.

($\Rightarrow$) Suppose that $m|n$. If $a \in \langle n \rangle$, then $n|a$. We have by Lemma 1(b) that $m|a$. This means that $a \in \langle m \rangle$. We have proved $\langle n \rangle \subseteq \langle m \rangle$

($\Leftarrow$) Suppose $\langle n \rangle \subseteq \langle m \rangle$. Because $n \in \langle n \rangle \subseteq \langle m \rangle$, we have $m|n$.

4. We have that $\gcd(2016, 323) = 1$, because

$$\begin{aligned} 2016 &= 6 * 323 + 78 \\ 323 &= 4 * 78 + 11 \\ 78 &= 7 * 11 + 1 \\ 11 &= 11 * 1 + 0 \end{aligned}$$

We can now write

$$\begin{aligned} 1 &= 78 - 7 * 11 = (2016 - 6 * 323) - 7 * (323 - 4 * 78) \\ &= 2016 - 13 * 323 + 28 * 78 = 2016 - 13 * 323 + 28 * (2016 - 6 * 323) \\ &= 29 * 2016 - (13 + 28 * 6) * 323 = \boxed{29} * 2016 - \boxed{181} * 323 \end{aligned}$$

5. (a) $\text{lcm}(8,12) = 24,\ \ \text{lcm}(20,30) = 60,\ \ \text{lcm}(51,68) = 204,\ \ \text{lcm}(23,18) = 414$

(b) For instance, $\gcd(51,68) = 17$ and $\text{lcm}(51,68) = 204$.
Now $51 * 68 = 3468$ and $17 * 204 = 3468$. It seems to be that

$$a * b = \text{lcm}(a,b) * \gcd(a,b)$$

(c) By (b), we have that

$$\text{lcm}(a,b) = \frac{a * b}{\gcd(a,b)}$$

We have $\gcd(301337, 307829) = 541$, because

$$\begin{aligned} 301337 &= 0 * 307829 + 301337 \\ 307829 &= 1 * 301337 + 6492 \\ 301337 &= 46 * 6492 + 2705 \\ 6492 &= 2 * 2705 + 1082 \\ 2705 &= 2 * 1082 + 541 \\ 1082 &= 2 * 541 + 0 \end{aligned}$$

We can now solve

$$\text{lcm}(301337, 307829) = (301337 * 307829)/541 = 171\,460\,753$$

6. First we see that

$$\frac{ab}{\gcd(a,b)} = a\frac{b}{\gcd(a,b)} = b\frac{a}{\gcd(a,b)}$$

This means that $\frac{ab}{\gcd(a,b)}$ is a common multiple of a and b. Because $\text{lcm}(a,b)$ is the smallest common multiple of a and b, we have

$$\frac{ab}{\gcd(a,b)} \geq \text{lcm}(a,b) \tag{1}$$

On the other hand, by Theorem 2 (Division Theorem), we can write

$$ab = q\,\text{lcm}(a,b) + r, \text{ where } 0 \leq r < \text{lcm}(a,b).$$

Because $\text{lcm}(a,b) = sa$ and $\text{lcm}(a,b) = tb$ for some s and t, we have $ab = qsa + r$. If we divide by a, we get $b = qs + \frac{r}{a}$. Similarly, we have $ab = qtb + r$ and dividing by b we obtain $a = qt + \frac{r}{b}$. Suppose that $r \neq 0$. Then the above mean that $a|r$ and $b|r$. Therefore, there are k_1 and k_2 such that $r = k_1 a = k_2 b$, and r is a common multiplier of a and b. On the other hand $r < \text{lcm}(a,b)$, which contradicts the minimality of $\text{lcm}(a,b)$. Hence, we must have $r = 0$ and $\text{lcm}(a,b)$ divides ab. Notice that

$$\frac{ab}{\text{lcm}(a,b)} = \frac{a}{\text{lcm}(a,b)/b} = \frac{b}{\text{lcm}(a,b)/a}$$

is a common divisor of a and b. By the maximality of the $\gcd(a, b)$,

$$\frac{ab}{\operatorname{lcm}(a, b)} \leq \gcd(a, b),$$

which directly gives

$$\frac{ab}{\gcd(a, b)} \leq \operatorname{lcm}(a, b) \tag{2}$$

Combining (1) and (2), we get

$$ab = \operatorname{lcm}(a, b) \gcd(a, b)$$