1. One way to start is to see that $8 = 2^3$. Then we may observe that $343^{\frac{1}{3}} = 7$, that is, $7^3 = 343$. We can write:

$$\left(\frac{8}{343}\right)^{-\frac{2}{3}} = \left(\frac{2^3}{7^3}\right)^{-\frac{2}{3}} = \left(\frac{2}{7}\right)^{-3\cdot\frac{2}{3}} = \left(\frac{2}{7}\right)^{-2}.$$

We know that for all $x \in \mathbb{R}$,

$$x^{-2} = \left(\frac{1}{x}\right)^2.$$

Therefore,

$$\left(\frac{2}{7}\right)^{-2} = \left(\frac{7}{2}\right)^2 = \frac{49}{4}.$$

2. (a) (Case i) If $x \geq 1/4$, then $|4x-1| = 4x-1$. We have the solution.

$$4x - 1 = 3 \iff 4x = 4 \iff x = 1.$$

Now the solution $x = 1$ is in the right area $x \geq 1/4$
(Case ii) If $x < 1/4$, then $|4x-1| = 1-4x$. We have the solution

$$1 - 4x = 3 \iff 4x = -2 \iff x = -\frac{1}{2}.$$

Also now the solution $x = -\frac{1}{2}$ is in the right area $x < 1/4$

(b) (Case i) If $x \geq -2$, then $|x+2| = x+2$. We get

$$x + 2 = \frac{1}{3}x + 5 \iff x - \frac{1}{3}x = 5 - 2 \iff \frac{2}{3}x = 3 \iff x = \frac{9}{2} = 4\frac{1}{2}.$$

The solution belongs to the area.
(Case ii) If $x < -2$, then $|x+2| = -x-2$. The solution is

$$-x - 2 = \frac{1}{3}x + 5 \iff x + \frac{1}{3}x = -7 \iff \frac{4}{3}x = -7 = x = -\frac{7 \cdot 3}{4} = -\frac{21}{4}.$$

The solution belongs to the area.

3. (Case i) If $x \geq 5/6$, then $|6x-5| = 6x-5$ and $|3x+4| = 3x+4$. The solution is

$$6x - 5 = 3x + 4 \iff x = 3$$

The solution belongs to the area.
(Case ii) If $-3/4 \leq x < 5/6$, then $|6x-5| = 5-6x$ and $|3x+4| = 3x+4$. We can solve:

$$5 - 6x = 3x + 4 \iff x = \frac{1}{9}.$$

The solution belongs to the area.

(Case iii) if $x < -3/4$, then $|6x - 5| = 5 - 6x$ and $|3x + 4| = -3x - 4$. The solution is

$$5 - 6x = -3x - 4 \iff x = 3.$$

The solution does no belong to the area. But no worries, because we already have found this solution.

4. Let A, B, and C be sets and suppose that there are bijections $f \colon A \to B$ and $g \colon B \to C$.
(i) We prove that $g \circ f \colon A \to C$ is a surjection. Let $c \in C$. Because g is a bijection, it is a surjection. Therefore, there is $b \in B$ such that $g(b) = c$. Moreover, because f is surjective, there is $a \in A$ such that $f(a) = b$. We have that

$$(g \circ f)(a) = g(f(a)) = g(b) = c.$$

This means that $g \circ f$ is a surjection.

(ii) We prove that $g \circ f \colon A \to C$ is an injection. Suppose that $(g \circ f)(a) = (g \circ f)(b)$ for some $a, b \in A$, that is, $g(f(a)) = g(f(b))$. Because g is a injection, we have $f(a) = f(b)$. Because f is an injection, we get $a = b$.

Since $g \circ f$ is both surjective and injective, it is a bijection.

5. The solution is that each guest moves to the next room: guest in room 1 moves to room 2, guest in room 2 moves to room 3, guest in room 3 moves to room 4, and so on. Because the hotel is enumerable infinite, this can be done. The new guest can enter to room 1.

6. The solution is that the guest in room k moves to the room $2k$. This means that all rooms with odd number get free: 1, 3, 5, 7, ... There are countable infinite number of new passengers:

$$x_0, x_1, x_2, x_3, \ldots$$

We can accommodate them in rooms having odds numbers by the rule (function) that the guest x_k goes to the room $2k + 1$.