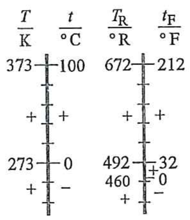
TEMPERATURE SCALES



Numerical value equations

$\{T\}_K = \{t\}_{°C} + 273{,}15$ 1

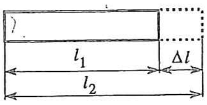
$\{T\}_K = \frac{5}{9}\left(\{t_F\}_{°F} - 32\right) + 273{,}15$ 2

$\{t\}_{°C} = \frac{5}{9}\left(\{t_F\}_{°C} - 32\right)$ 3

$\{T_R\}_{°R} = \{t_F\}_{°F} + 459{,}67 \approx \{t_F\}_{°F} + 460$ 4

T = thermodynamic temperature, $[T] = K$
$T_0 = 273{,}15$ K (≈ 273 K)
t = celsius temperature, $[t] = °C$
t_F = fahrenheit temperature, $[t_F] = °F$
T_R = rankine temperature, $[T_R] = °R$

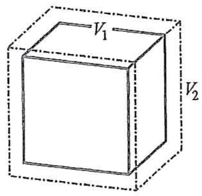
THERMAL EXPANSION



The change in length $\Delta l = l_2 - l_1$

$\Delta l = \alpha l_1 \Delta t$ 5

α = linear expansion coefficient, $[\alpha] = 1/°C = 1/K$
l_1 = initial length
$\Delta t = t_2 - t_1$ = the change in temperature

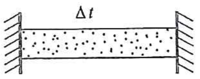


The change in volume $\Delta V = V_2 - V_1$

$\Delta V = \gamma V_1 \Delta t$ 6

γ = cubic expansion coefficient, $[\gamma] = 1/°C = 1/K$
V_1 = initial volume
$\Delta t = t_2 - t_1$ = the change in temperature

For solids $\gamma = 3\alpha$ 7



The change in length is prevented

Thermal stress

$\sigma = -E\alpha\Delta t$ 8

$\sigma = \frac{F}{A}$ = stress, $[\sigma] = \frac{N}{m^2} = Pa$
E = Young's modulus, $[E] = Pa$

EQUATION OF STATES FOR IDEAL GASES

$pV = nRT$ — p = pressure, $[p] = Pa$ 1

$n = \frac{m}{M}$ — V = volume, $[V] = m^3$ 2

$\rho = \frac{pM}{RT}$ — T = thermodynamic temperature; n = number of moles, $[n] = mol$; m = mass; ρ = density 3

$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ — M = molar mass, $[M] = kg/mol$ 4

R = molar gas constant $= 8{,}3145\frac{J}{mol\,K} = 0{,}083145\frac{dm^3\,bar}{mol\,K} \left(= 0{,}8479\frac{kpm}{mol\,K}\right)$

QUANTITY OF HEAT Q $[Q] = J$ 1 $cal_{IT} = 4{,}1868$ J

$Q = mc\Delta T$	c = specific heat (capacity)	$[c] = \frac{J}{kg°C}$	5
$Q = C\Delta T$, $C = m_1c_1 + m_2c_2 + ...$	C = heat capacity	$[C] = \frac{J}{°C}$	6
$Q = ms$	s = heat of fusion	$[s] = J/kg$	7
$Q = mr$	r = heat of vapourization	$[r] = J/kg$	8

Specific heat capacities of a gas at constant volume and at constant pressure

$c_V = \frac{1}{m}\left(\frac{dQ}{dT}\right)_V$ $\qquad c_p = \frac{1}{m}\left(\frac{dQ}{dT}\right)_p$

Molar heat capacities of a gas at constant volume and at constant pressure

$C_{mV} = Mc_V$ $\qquad C_{mp} = Mc_p$ $\qquad [C_m] = \frac{J}{mol\,K}$ 11

ratio of the specific heat capacities $\gamma = \frac{c_p}{c_V} = \frac{C_{mp}}{C_{mV}}$ $\qquad$ gas constant $R = C_{mp} - C_{mV}$ 12

Monatomic gases $\gamma \approx 1{,}67$ $\qquad$ Diatomic gases $\gamma \approx 1{,}40$

HUMIDITY OF AIR

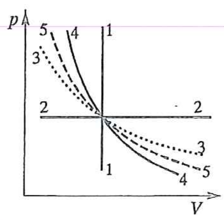
$\varphi = 100 \cdot \frac{p_v}{p_{vs}}\,\%$ 13

$\rho_v = \frac{p_v M}{RT}$ 14

φ = relative humidity
p_v = partial pressure of water vapour
p_{vs} = saturated vapour pressure
ρ_v = density of water vapour, $[\rho_v] = kg/m^3$
$M = 18 \cdot 10^{-3}$ kg/mol (water)

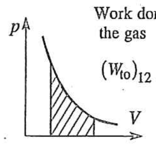
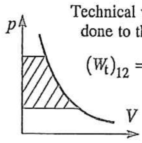
IDEAL GAS PROCESSES

Process	Equations of states for the process	Heat absorbed in the process Q	
Isochoric (volume $= V_c$ = constant)	$\frac{p_1}{p_2} = \frac{T_1}{T_2}$	$Q = mc_V(T_2 - T_1)$	1
Isobaric (pressure $= p_c$ = constant)	$\frac{V_1}{V_2} = \frac{T_1}{T_2}$	$Q = mc_p(T_2 - T_1)$	2
Isothermic (temperature $= T_c$ = constant)	$p_1V_1 = p_2V_2$	$Q = \frac{m}{M}RT_c\ln\frac{p_1}{p_2}$ $= p_1V_1\ln\frac{V_2}{V_1}$	3
Isentropic or adiabatic (if no heat transfer, then n $= \gamma$)	$p_1V_1^n = p_2V_2^n$ $\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{1/n}$	$Q = 0$	4
Polytropic n = polytropic index or exponent 1 < n < γ	$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$ $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1}$	$Q = m\frac{\gamma - n}{1-n}c_V(T_2 - T_1)$ $= mc_nT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$ $= c_n\frac{Mp_1V_1}{R}\left(\frac{T_2}{T_1} - 1\right)$	5



p, *V*-diagrams of processes

1 isochoric
2 isobaric
3 isothermic
4 isentropic (adiabatic)
5 polytropic

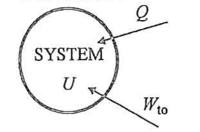
Work done to the gas	Technical work done to the gas	Change in entropy	
 $(W_{to})_{12} = -\int_1^2 p\,dV$	 $(W_t)_{12} = \int_1^2 V\,dp$	$\Delta S = \int_1^2 \frac{dQ}{T}$	
$(W_{to})_{12} = 0$	$(W_t)_{12} = V_c(p_2 - p_1)$	$\Delta S = mc_V\ln\frac{T_2}{T_1}$	1
$(W_{to})_{12} = p_c(V_1 - V_2)$	$(W_t)_{12} = 0$	$\Delta S = mc_p\ln\frac{T_2}{T_1}$	2
$(W_{to})_{12} = -Q$	$(W_t)_{12} = -Q$	$\Delta S = \frac{mR}{M}\ln\frac{p_1}{p_2}$ $\Delta S = \frac{p_1V_1}{T_c}\ln\frac{V_2}{V_1}$	3
$(W_{to})_{12} = \frac{mR}{M(n-1)}(T_2 - T_1)$ $= \frac{p_1V_1}{n-1}\left(\frac{T_2}{T_1} - 1\right)$ $= \frac{mRT_1}{M(n-1)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$ $= \frac{p_1V_1}{n-1}\left[\left(\frac{V_1}{V_2}\right)^{n-1} - 1\right]$		$\Delta S = 0$	4
	$(W_t)_{12} = n(W_{to})_{12}$	$\Delta S = mc_n\ln\frac{T_2}{T_1}$	5

NOTES
Heat absorbed by the gas is positive.
Work done by the gas is negative.
Technical work done by the gas is negative
Specific gas constant $R_x = \frac{R}{M_x}$; $[R_x] = \frac{J}{kg\,K}$.
Specific values of quantities are obtained by dividing the above values by mass.
$c_n = \frac{\gamma - n}{1 - n} \cdot c_V$ = polytropic specific heat capacity (< 0).

LAWS OF THERMODYNAMICS

I LAW (ENERGY IS CONSERVED) or (ENERGY PRINCIPLE)

$\Delta U = Q + W_{to}$ — The change of internal energy is the sum of heat absorbed by the system and work done to the system 1



$\Delta U = m c_V \Delta T$ = the change of internal energy

Q = heat absorbed

$W_u = -\int_1^2 p\,dV$ = work done to the system

$\Delta I = m c_p \Delta T$ = change in enthalpy 2

Using the pV term

$\Delta I = \Delta U + (p_2 V_2 - p_1 V_1)$ 3

II LAW (ENTROPY PRINCIPLE)

There is a tendency on the part of nature to proceed toward a state of greater disorder. 4

The total entropy S of the system plus environment increases in any natural process. $[S]$ = J/K

III LAW (ABSOLUTE ZERO PRINCIPLE)

It is impossible by any procedure to reduce any system to the absolute zero of temperature.

KINETIC THEORY OF GASES

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3p}{\rho}}$$

v_{rms} = root mean square speed of gas molecules = effective speed 5

$$E = \frac{1}{2} kT$$

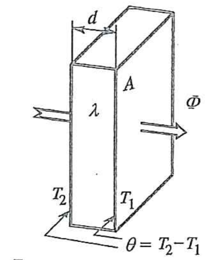
E = energy per degree of freedom 6

$k = 1{,}3807 \cdot 10^{-23} \dfrac{\text{J}}{\text{K} \cdot \text{(molecule)}}$ = Boltzmann's constant

HEAT TRANSFER

$\Phi = \dot{Q} = \dfrac{dQ}{dt}$ = heat flow rate $\quad [\Phi] = \dfrac{\text{J}}{\text{s}} = \text{W}$ 1

$q = \dfrac{\Phi}{A}$ = density of heat flow rate $\quad [q] = \dfrac{\text{W}}{\text{m}^2}$ 2



CONVECTION

$\Phi = q_m c \Delta T$ 3

q_m = mass flow rate $\quad [q_m] = \text{kg/s}$

CONDUCTION THROUGH A WALL

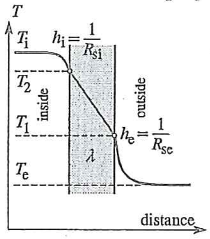
$$\Phi = \lambda A \frac{\theta}{d} = \frac{A\theta}{R}$$ 4

λ = thermal conductivity $\quad [\lambda] = \dfrac{\text{W}}{\text{m} \cdot \text{K}}$

θ = temperature difference

d = thickness of the layer

$R = \dfrac{d}{\lambda}$ = thermal resistance of the layer $\quad [R] = \dfrac{\text{m}^2\,\text{K}}{\text{W}}$



TRANSFER AT A SURFACE

$\Phi = h_i A (T_i - T_2) = h_e A (T_1 - T_e)$ 5

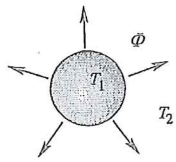
h = coefficient of heat transfer $\quad [h] = \dfrac{\text{W}}{\text{m}^2 \cdot \text{K}}$

TRANSFER THROUGH A COMPOSITE WALL

$\theta = T_i - T_e$

$$\Phi = \frac{A\theta}{R_T} \qquad R_T = \frac{1}{U} = R_{se} + \frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \ldots + R_{si}$$ 6

U-coefficient and thermal resistance R_T of the wall



RADIATION

Stefan-Boltzmann law

$\Phi = \varepsilon \sigma A T^4$ 7

ε = emissivity of the surface

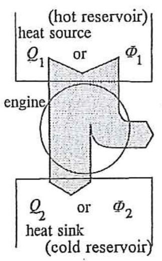
$\sigma = 5{,}6705 \cdot 10^{-8} \dfrac{\text{W}}{\text{m}^2\text{K}^4}$ = Stefan - Boltzmann constant

T = absolute temperature of the radiating surface (area A)

The heat flow rate from a surface (T_1) to its surroundings (T_2)

$\Phi = \varepsilon \sigma A (T_1^4 - T_2^4)$ 8

HEAT ENGINES



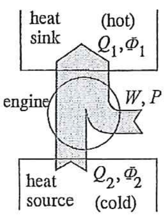
Heat engine: thermal efficiency

$$\eta = \frac{Q_1 - |Q_2|}{Q_1} = \frac{|W|}{Q_1} = \frac{\Phi_1 - |\Phi_2|}{\Phi_1} = \frac{|P|}{\Phi_1}$$ 1

Q_1, Φ_1 = absorbed heat or heat flow

Q_2, Φ_2 = rejected heat or heat flow

W, P = delivered work or power



Refrigerator or heat pump

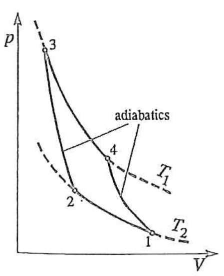
coefficient of performance (COP) in refrigeration

$$\varepsilon_{refr} = \frac{Q_2}{W} = \frac{Q_2}{|Q_1| - Q_2} = \frac{\Phi_2}{P}$$ 2

coefficient of performance (COP) in heating

$$\varepsilon_{heat} = \frac{|Q_1|}{W} = \frac{|Q_1|}{|Q_1| - Q_2} = \frac{|\Phi_1|}{P} = \varepsilon_{refr} + 1$$ 3

IDEAL CARNOT CYCLES (CONSTANT TEMPERATURE CYCLES)



Heat engine: $1 \to 2 \to 3 \to 4 \to 1$

Carnot efficiency η_C

$$\eta_C = \frac{T_1 - T_2}{T_1}$$ 4

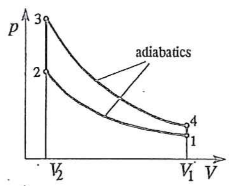
Refrigerator and heat pump: $1 \to 4 \to 3 \to 2 \to 1$

Carnot COP in refrigeration

$$\varepsilon_{C,refr} = \frac{T_2}{T_1 - T_2}$$ 5

Carnot COP in heating

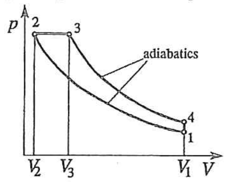
$$\varepsilon_{C,heat} = \frac{T_1}{T_1 - T_2}$$ 6



OTTO CYCLE (constant volume cycle)

$$\eta = 1 - \frac{1}{\varepsilon^{\gamma-1}} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{T_1}{T_2}$$ 1

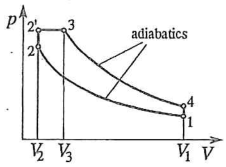
$\varepsilon = \dfrac{V_1}{V_2}$ = compression ratio



DIESEL CYCLE

$$\eta = 1 - \frac{1}{\varepsilon^{\gamma-1}} \cdot \frac{\varphi^{\gamma} - 1}{\gamma(\varphi - 1)}$$ 2

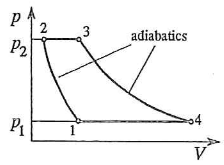
$\varphi = \dfrac{V_3}{V_2}$ = cut - off ratio



DUAL COMBUSTION CYCLE

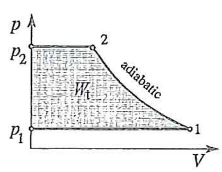
$$\eta = 1 - \frac{1}{\varepsilon^{\gamma-1}} \cdot \frac{\psi \varphi^{\gamma} - 1}{\psi - 1 + \psi\gamma(\varphi - 1)}$$ 3

$\psi = \dfrac{p_{2'}}{p_2}$



GAS TURBINE CYCLE (constant pressure cycle)

$$\eta = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{T_1}{T_2}$$ 4

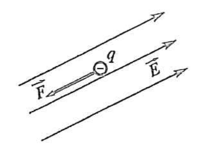


COMPRESSOR CYCLE

$$W_t = \frac{\gamma}{\gamma - 1}(p_2 V_2 - p_1 V_1) = \frac{\gamma p_1 V_1}{\gamma - 1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \frac{\gamma n R}{\gamma - 1}(T_2 - T_1)$$ 5

W_t = actual work done

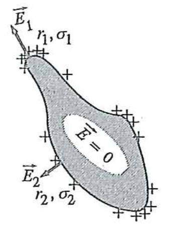
n = number of gas moles



ELECTRIC FIELD

$\vec{E} = \dfrac{\vec{F}}{q}$ $\qquad [E] = \dfrac{\text{N}}{\text{As}} = \dfrac{\text{V}}{\text{m}}$ 1

F = force exerted on a charge q by an electric field E



At the outer surface of a solid conductor

$\dfrac{\sigma_1}{\sigma_2} \approx \dfrac{r_2}{r_1}$ 2

σ = surface charge density $\quad [\sigma] = \text{As}/\text{m}^2$

r = radius of curvature of the surface

Immediately outside a charged conductor

$E = \dfrac{\sigma}{\varepsilon_r \varepsilon_0}$ 3

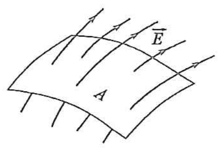
ε_0 = permittivity of vacuum $\approx 8{,}8542 \cdot 10^{-12} \dfrac{\text{A}^2\text{s}^2}{\text{Nm}^2}$

ε_r = relative permittivity

Inside the conductor $\quad E = 0$

POTENTIAL V

is constant inside a solid conductor = at the surface



ELECTRIC FLUX through the area A

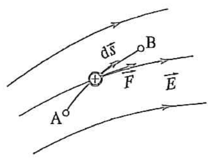
$\Psi = \int_A \vec{D} \cdot \mathrm{d}\vec{A}$ $\qquad [\Psi] = \text{As}$ 4

$D = \varepsilon_r \varepsilon_0 E$ = electric flux density, $[D] = \dfrac{\text{As}}{\text{m}^2}$

Gauss's law

$\Psi = \oint_A \vec{D} \cdot \mathrm{d}\vec{A} = Q$ 5

Q = charge inside a closed surface A

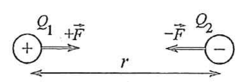


POTENTIAL DIFFERENCE $V_A - V_B$ i.e. VOLTAGE U_{AB}

$U_{AB} = V_A - V_B$

$= \int_A^B \vec{E} \cdot \mathrm{d}\vec{s} = \dfrac{W_{A \to B}}{q} = \dfrac{\int_A^B \vec{F} \cdot \mathrm{d}\vec{s}}{q}$ 6

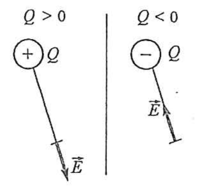
$[V] = [U] = \text{V} = \dfrac{\text{J}}{\text{As}}$



Coulomb's law

The force between two point charges is

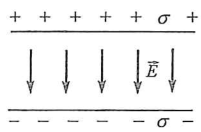
$F = \dfrac{1}{4\pi\varepsilon_r\varepsilon_0} \dfrac{Q_1 Q_2}{r^2}$ 1



Point charge

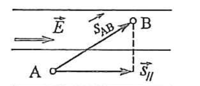
$E = \dfrac{1}{4\pi\varepsilon_r\varepsilon_0} \dfrac{Q}{r^2}$ 2

$V = \dfrac{1}{4\pi\varepsilon_r\varepsilon_0} \dfrac{Q}{r}$ $\qquad$ reference potential $V = 0$ at $r = \infty$. 3



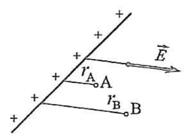
Between two large parallel planes is **a uniform electric field**

$E = \dfrac{\sigma}{\varepsilon_r \varepsilon_0}$ 4



In a uniform electric field

$U_{AB} = \vec{E} \cdot \vec{s}_{AB} = E s_{//}$ 5



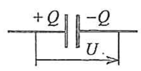
Line charge

$E = \dfrac{1}{2\pi\varepsilon_r\varepsilon_0} \dfrac{\lambda}{r}$ 6

$U_{AB} = \dfrac{\lambda}{2\pi\varepsilon_r\varepsilon_0} \ln \dfrac{r_B}{r_A}$ 7

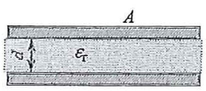
λ = linear charge density, $[\lambda] = \dfrac{\text{As}}{\text{m}}$

CAPACITORS



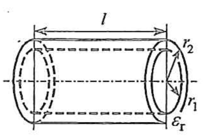
Capacitance

$C = \dfrac{Q}{U}$ $\qquad [C] = \dfrac{\text{As}}{\text{V}} = \text{F}$ 1



Parallel-plate capacitor

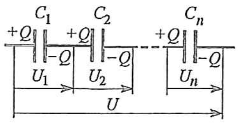
$C = \varepsilon_r \varepsilon_0 \dfrac{A}{d}$ $\qquad E = \dfrac{U}{d}$ 2



Cylindrical capacitor

$C = 2\pi\varepsilon_r\varepsilon_0 \dfrac{l}{\ln \frac{r_2}{r_1}}$ $\qquad E = \dfrac{U}{\ln \frac{r_2}{r_1}} \cdot \dfrac{1}{r}$ 3

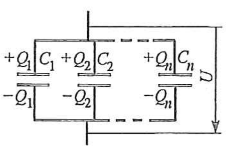
$r_1 \le r \le r_2$



Capacitors in series

$\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots + \dfrac{1}{C_n}$ 4

$U = U_1 + U_2 + \ldots + U_n$



Capacitors in parallel

$C = C_1 + C_2 + \ldots + C_n$ 5

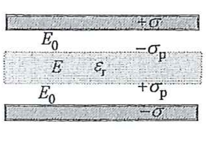
$Q = Q_1 + Q_2 + \ldots + Q_n$

Energy stored in the capacitor

$W = \dfrac{1}{2} C U^2 = \dfrac{1}{2} Q U = \dfrac{1}{2} \dfrac{Q^2}{C}$ 6

Energy density of the electric field

$w = \dfrac{1}{2} \varepsilon_r \varepsilon_0 E^2 = \dfrac{1}{2} D E$ 7



Induced surface charge by polarization

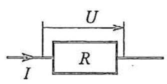
$\sigma_p = \left(1 - \dfrac{1}{\varepsilon_r}\right) \sigma$ 8

Electric field within the dielectric

$E = \dfrac{E_0}{\varepsilon_r}$, external field $E_0 = \dfrac{\sigma}{\varepsilon_0}$ 9

DIRECT CURRENT

$I = \dfrac{\mathrm{d}Q}{\mathrm{d}t}$ instantaneous current and $I = \dfrac{Q}{t}$, if I is constant $\qquad [I] = \text{A}$ 1



OHM'S LAW $\qquad U = IR$ 2

Resistance $\qquad R = \rho \dfrac{l}{A}$ 3

ρ = resistivity, $[\rho] = \Omega\text{mm}^2/\text{m}$, Ωm

Resistivity and temperature (given temperature interval)

Metal conductor

$\rho = \rho_0 \cdot (1 + \alpha_R \Delta T)$

α_R = temperature coefficient of resistivity

$[\alpha_R] = 1/\text{K}$

$\Delta T = T - T_0$

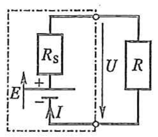
NTC-resistor

$\rho = \rho_\infty e^{B/T}$

ρ_∞ and B constants for the material

T = absolute temperature

DIRECT CURRENT CIRCUIT



Source of energy (emf source)

$E = U + IR_s$ 5

E = electromotive force

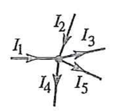
U = terminal voltage

R_s = internal resistance

Power

$P = UI = I^2 R = \dfrac{U^2}{R}$ 6

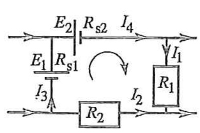
Kirchhoff's rules



Junction rule: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction. 7

The junction shown in the figure

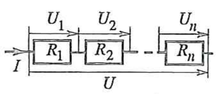
$I_1 + I_2 = I_3 + I_4 + I_5$



Loop rule: Around any closed loop the algebraic sum of emfs is equal to the algebraic sum of all the potential drops in the resistances of that loop. 8

The loop shown in the figure

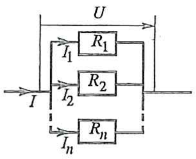
$E_1 - E_2 = I_3 R_{s1} + I_4 R_{s2} + I_1 R_1 - I_2 R_2$



Resistors in series

$R = R_1 + R_2 + ... + R_n$

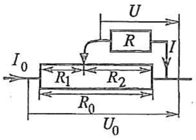
$U = U_1 + U_2 + ... + U_n$ and $U_i = IR_i$ 1



Resistors in parallel

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$

$I = I_1 + I_2 + ... + I_n$ and $I_i = \frac{U}{R_i}$ 2

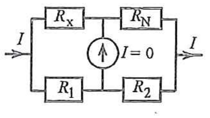


POTENTIAL DIVIDER, potentiometer

$U = \frac{RR_0 - RR_1}{R_1R_0 + RR_0 - R_1^2} \cdot U_0$

$I = \frac{R_0 - R_1}{R_0 + R - R_1} \cdot I_0$ 3

If $R >> R_0$, then $U = \frac{R_2}{R_0} \cdot U_0$ and $I = 0$.

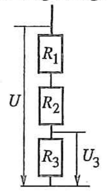


WHEATSTONE BRIDGE CIRCUIT

In balance $I = 0$. Then

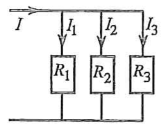
$\frac{R_x}{R_N} = \frac{R_1}{R_2}$ 4

Dividing voltage



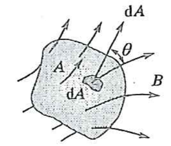
$U_3 = \frac{R_3}{R_1 + R_2 + R_3} \cdot U$

Dividing current



$I_2 = \frac{1/R_2}{1/R_1 + 1/R_2 + 1/R_3} \cdot I$

MAGNETIC FIELD

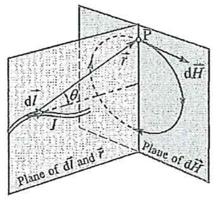


$\Phi = \int_A \vec{B} \cdot d\vec{A}$ 1

If $\vec{B}$ is constant and $// \vec{A}$, then $\Phi = BA$

Φ = magnetic flux, $[\Phi] = \text{Vs} = \text{Wb}$

B = magnetic flux density, = magnetic field $[B] = \frac{\text{Vs}}{\text{m}^2} = \text{T}$



$\vec{B} = \mu_r \mu_0 \vec{H}$ 2

μ_0 = permeability of vacuum $= 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$

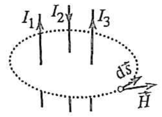
μ_r = relative permeability

H = magnetic field strength, $[H] = \frac{\text{A}}{\text{m}}$

Biot-Savart's law

$d\vec{H} = \frac{I}{4\pi} \cdot \frac{d\vec{l} \times \vec{r}^0}{r^2}$ $\quad d\vec{H} \perp \vec{r} \wedge d\vec{H} \perp d\vec{l}$ 3

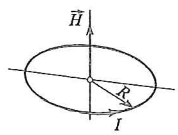
$dH = \frac{I}{4\pi} \cdot \frac{dl \sin\theta}{r^2}$



Ampère's law

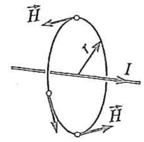
$\oint_s \vec{H} \cdot d\vec{s} = \Sigma I$ 4

In the figure $\sum I = I_1 - I_2 + I_3$



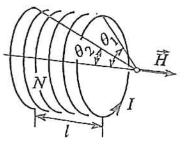
At the centre of a circular ring

$H = \frac{I}{2R}$



Field of a long straight wire

$H = \frac{I}{2\pi r}$



Along the axis of a solenoid

$H = \frac{NI}{2l}(\cos\theta_1 - \cos\theta_2)$ 5

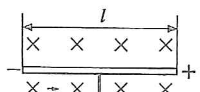
At the centre of a long solenoid

$H = \frac{NI}{l}$

ELECTROMAGNETIC INDUCTION

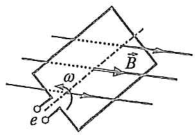
Faraday's law of induction

$E = -N\frac{d\Phi}{dt}$ $\quad \frac{d\Phi}{dt}$ = the time rate change of magnetic flux 1



A straight conductor moves in magnetic field $B \Rightarrow$

$E = vlB$ $\quad \vec{v} \perp \vec{B}$ 2



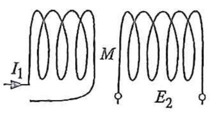
A coil rotating in magnetic field $\Rightarrow$ sinusoidal voltage

$e = NAB\omega \sin \omega t$ 3

ω = angular velocity $= 2\pi f$ (f = frequency)

N = number of turns

A = coil area



Mutual inductance $M \Rightarrow$ emf in coil 2

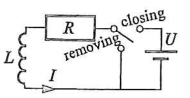
$|E_2| = M\frac{|dI_1|}{dt}$ $\quad [M] = \text{H} = \frac{\text{Vs}}{\text{A}}$ 4

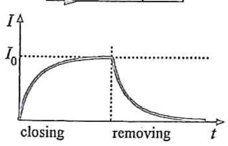
Self inductance L of a coil $\Rightarrow$ self induced emf

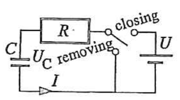
$|E| = L\frac{|dI|}{dt}$ $\quad [L] = \text{H} = \frac{\text{Vs}}{\text{A}}$ 5

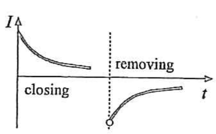
For a solenoid $L = \frac{N\Phi}{I} \approx \frac{N^2 \mu_r \mu_0 A}{l}$

Energy stored in the inductor $W_m = \frac{1}{2}LI^2$ $\quad$ Energy density in the magnetic field $w_m = \frac{1}{2}HB = \frac{1}{2}\mu_r\mu_0 H^2 = \frac{1}{2}\frac{B^2}{\mu_r\mu_0}$ 6









RL-circuit time constant $\tau = L / R$

Closing $I = \frac{U}{R}\left(1 - e^{-t/\tau}\right)$

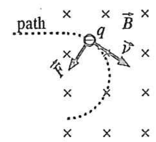
Removing $I = I_0 e^{-t/\tau}$

RC-circuit time constant $\tau = RC$

In closing, if $U_{C0} = 0$ $\quad U_C = U\left(1 - e^{-t/\tau}\right)$ ja $I = \frac{U}{R}e^{-t/\tau}$ 7

Removing $U_C = U_{C0}\, e^{-t/\tau}$ ja $I = -\frac{U_{C0}}{R}e^{-t/\tau}$ 8

MAGNETIC FORCES

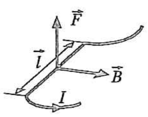


On a moving charge

$\vec{F} = q\vec{v} \times \vec{B}$ $\quad \vec{F} \perp \vec{v} \wedge \vec{F} \perp \vec{B}$ 1

$F = |q| vB \sin\alpha$

If $\vec{v} \perp \vec{B} \Rightarrow$ circular orbit with $r = \frac{mv}{qB}$

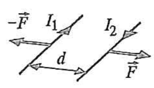


On a current-carrying conductor

$\vec{F} = I\vec{l} \times \vec{B}$ $\quad \vec{F} \perp \vec{l} \wedge \vec{F} \perp \vec{B}$ 2

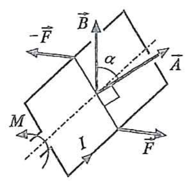
$F = IlB \sin\alpha$

If $\vec{l} \perp \vec{B} \Rightarrow F = IlB$



The force between two parallel wires

$F = \frac{\mu_r \mu_0 I_1 I_2 l}{2\pi d}$ 3



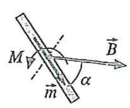
On a current-carrying loop

$\vec{M} = NI\vec{A} \times \vec{B}$ 4

$M = NIAB \sin\alpha$

Magnetic moment of the loop

$\vec{m} = NI\vec{A}$ $\quad [m] = \text{Am}^2$

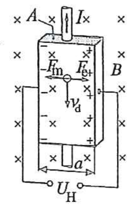


On a bar magnet

$\vec{M} = \vec{m} \times \vec{B}$ 5

$M = mB \sin\alpha$

m = magnetic moment of the magnet



THE HALL VOLTAGE

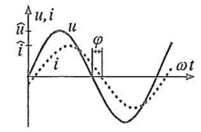
$U_H = \frac{IBa}{neA}$, $\quad I = neAv_d$ 6

n = density of charge carriers, $[n] = 1/\text{m}^3$

A = cross-sectional area, $[A] = \text{m}^2$

v_d = drift velocity, $[v_d] = \text{m/s}$

ALTERNATING CURRENT (Sinusoidal)

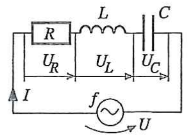


$u = \hat{u}\sin\omega t$ $\quad U = \hat{u}/\sqrt{2}$ 1

$i = \hat{i}\sin(\omega t - \varphi)$ $\quad I = \hat{i}/\sqrt{2}$

Ohm's law in the AC circuit

$U = IZ$ 2



u, i = istantaneous values of voltage and current

$\hat{u}, \hat{i}$ = maximum values

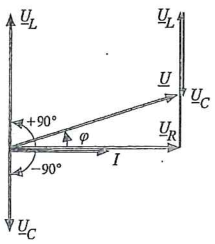
U, I = effective values (rms values)

$\omega = 2\pi f$ = angular frequency

$f = 1/T$ = frequency

T = period

φ = phase difference between voltage and current



$Z = \sqrt{R^2 + X^2}$ = impedance

R = resistance

$X = X_L - X_C$ = reactance

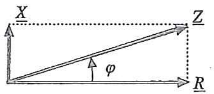
$X_L = \omega L$ = inductive reactance

$X_C = \dfrac{1}{\omega C}$ = capacitive reactance

$\cos\varphi = \dfrac{R}{Z}$ = power factor

$U_R = IR, \quad U_C = IX_C, \quad U_L = IX_L$

$U = IZ$



$U^2 = U_R^2 + (U_L - U_C)^2$ 3

$\underline{U} = \underline{U}_R + \underline{U}_L + \underline{U}_C$

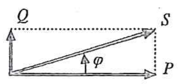
Powers in the AC circuits

Active power

$P = UI\cos\varphi = I^2R$ $\quad [P] = \mathrm{W}$ 4

Apparent power

$S = UI = I^2Z$ $\quad [S] = \mathrm{VA}$ 5



Reactive power

$Q = UI\sin\varphi = I^2X$ $\quad [Q] = \mathrm{var}$ 6

Power triangle

$S^2 = P^2 + Q^2$ ja $\cos\varphi = \dfrac{P}{S}$

Resonance means that the phase difference between voltage and current is zero.

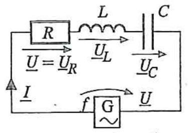
Series (voltage) resonance occurs in a RLC-circuit when $Z = R$, then

$U_L = X_L I = X_C I = U_C$.

The current I reaches its maximum value

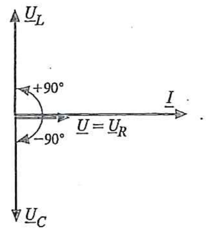
$I = \dfrac{U}{R}$.

The resonance frequency $f = \dfrac{1}{2\pi\sqrt{LC}}$



The situation in the phase diagram

$X_L = X_C = 2R$



Parallel (current) resonance occurs in a parallel connected circuit when

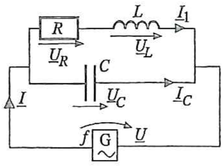
$R^2 + 4\pi^2 f^2 L^2 = \dfrac{L}{C}$.

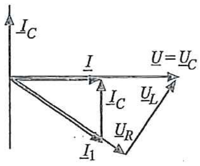
The current I is in its minimum value

$I = \dfrac{UR}{R^2 + 4\pi^2 f^2 L^2}$.

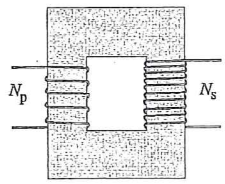
The resonance frequency

$f = \dfrac{1}{2\pi\sqrt{LC}} \cdot \sqrt{1 - \dfrac{R^2C}{L}}$





TRANSFORMER



In an ideal transformer

$\dfrac{U_{np}}{U_{ns}} = \dfrac{I_{ns}}{I_{np}} = \dfrac{N_p}{N_s}$

$S_n = U_{ns}I_{ns} = U_{np}I_{np}$

S_n = nominal power

n = nominal values

p = primary values

s = secondary values

Fundamental physical constants

Speed of light (in vacuum)	c	$2{,}9979 \cdot 10^{8}\ \mathrm{m \cdot s^{-1}}$
Gravitational constant	G	$6{,}6726 \cdot 10^{-11}\ \mathrm{N m^2 \cdot kg^{-2}}$
Charge of electron	e	$1{,}6022 \cdot 10^{-19}\ \mathrm{C}$
Electron rest mass	m_e	$9{,}1094 \cdot 10^{-31}\ \mathrm{kg}$
Proton rest mass	m_p	$1{,}6726 \cdot 10^{-27}\ \mathrm{kg}$
Neutron rest mass	m_n	$1{,}6749 \cdot 10^{-27}\ \mathrm{kg}$
Planck's constant	h	$6{,}6261 \cdot 10^{-34}\ \mathrm{J \cdot s} =$ $4{,}1357 \cdot 10^{-15}\ \mathrm{eVs}$
Rydberg constant	R_∞	$1{,}0974 \cdot 10^{7}\ \mathrm{m^{-1}}$
Avogadro's number	N_A	$6{,}0221 \cdot 10^{23}\ \mathrm{mol^{-1}}$
Boltzmann's constant	k	$1{,}3807 \cdot 10^{-23}\ \mathrm{J \cdot K^{-1}}$
Molar gas constant	R	$8{,}3145\ \mathrm{J/(mol \cdot K)}$
Permittivity of free space	ε_0	$8{,}8542 \cdot 10^{-12}\ \mathrm{C^2/(N m^2)}$
Permeability of free space	μ_0	$1{,}2566 \cdot 10^{-6}\ \mathrm{Vs/Am}$
Wien displacement constant		$2{,}8978 \cdot 10^{-3}\ \mathrm{m \cdot K}$
Stefan-Boltzmann's constant	σ	$5{,}6705 \cdot 10^{-8}\ \mathrm{W/(m^2 K^4)}$
Atomic mass unit	u	$1{,}6605 \cdot 10^{-27}\ \mathrm{kg}$
Energy equivalent of atomic mass unit	uc^2	931,49 MeV

The Greek Alphabet

alpha	Α	α	A	α	nu	Ν	ν	N	ν
beta	Β	β	B	β	xi	Ξ	ξ	Ξ	ξ
gamma	Γ	γ	Γ	γ	omicron	Ο	ο	O	o
delta	Δ	δ	Δ	δ	pi	Π	π, ϖ	Π	π, ϖ
epsilon	Ε	ε, ϵ	E	ε, ϵ	rho	Ρ	ϱ, ρ	P	ϱ, ρ
zeta	Ζ	ζ	Z	ζ	sigma	Σ	σ	Σ	σ
eta	Η	η	H	η	tau	Τ	τ	T	τ
theta	Θ	ϑ, θ	Θ	ϑ, θ	upsilon	Υ	υ	Y	υ
iota	Ι	ι	I	ι	phi	Φ	φ, ϕ	Φ	φ, ϕ
kappa	Κ	ϰ, κ	K	$\varkappa, \kappa$	chi	Χ	χ	X	χ
lambda	Λ	λ	Λ	λ	psi	Ψ	ψ	Ψ	ψ
mu	Μ	μ	M	μ	omega	Ω	ω	Ω	ω

PROPERTIES OF GASES

Gas		Molar mass $\frac{M}{10^{-3}\,\frac{kg}{mol}}$	Density at NTP $\frac{\rho_0}{\frac{kg}{m^3}}$	Boiling point (normal) $\frac{t_r}{°C}$	Latent heat of vaporization (normal) $\frac{r}{\frac{kJ}{kg}}$	Critical temperature $\frac{t_{kr}}{°C}$	Critical pressure $\frac{p_{kr}}{bar}$	Specific heat capacity at 0 °C $\frac{c_p}{\frac{kJ}{kg°C}}$	Ratio of the specific heat capacities γ
Air	(mixture)	29	1,293	−194	197	−141	37,7	1,001	1,40
Acetylene	C_2H_2	26	1,171	−84 [1]	829 [1]	+35,7	62,4	1,64	1,23
Ammonia	NH_3	17	0,771	−33,4	1369	+132	113	2,06	1,32
Carbon dioxide	CO_2	44	1,977	−74 [1]	574 [1]	+31	74,0	0,825	1,31
Helium	He	4	0,178	−269	20,9	−268	2,29	5,23	1,66
Hydrogen	H_2	2	0,090	−253	461	−240	13,0	14,24	1,41
Methane	CH_4	16	0,717	−162	548	−82,5	46,3	2,18	1,30
Nitrogen	N_2	28	1,251	−196	155	−147	33,9	1,043	1,40
Oxygen	O_2	32	1,29	−183	218	−119	50,4	0,913	1,40

[1] Sublimation point and heat

PROPERTIES OF LIQUIDS

Liquid	Molar mass $\frac{M}{10^{-3}\,\frac{kg}{mol}}$	Density at +20 °C $\frac{\rho}{\frac{kg}{m^3}}$	Volumic expansion coefficient $\frac{\gamma}{10^{-3}\,\frac{1}{°C}}$	Specific heat capacity 0 … 100 °C $\frac{c_p}{\frac{kJ}{kg°C}}$	Melting point $\frac{t_s}{°C}$	Latent heat of fusion $\frac{s}{\frac{kJ}{kg}}$	Normal boiling point $\frac{t_r}{°C}$	Latent heat of vaporization $\frac{r}{\frac{kJ}{kg}}$	Viscosity at +20 °C $\frac{\eta}{10^{-3}\,\frac{kg}{m \cdot s}}$	Surface tension at +20 °C $\frac{\sigma}{10^{-3}\,\frac{N}{m}}$	Modulus of compression $\frac{K}{10^9\,Pa}$
Benzene	78	871	1,2	1,76	+5,5	127	80,1	396	0,65	28,9	1,05
Carbon disulphide	76	1260	1,14	1,00	−112	74,1	46,3	356	0,37	32,3	
Ethanol	46	790	1,1	2,47	−115	105	78,3	854	1,20	22,3	1,1
Glycerol	92	1260	0,49	2,43	+18,0	199	290		1400	64	
Mercury	201	13550	0,182	0,140	−38,9	11,7	357	293	1,15	480	27
Water	18	998		4,19	0,0	333	100,0	2260	1,00	73	2,1

PROPERTIES OF SOLIDS

Substance	Density $\frac{\rho}{\frac{kg}{m^3}}$	Linear expansion coefficient $\frac{\alpha}{10^{-6}\,\frac{1}{°C}}$	Modulus of elasticity, Young's modulus $\frac{E}{10^9\,Pa}$	Shear modulus, modulus of rigidity $\frac{G}{10^9\,Pa}$	Specific heat capacity $\frac{c_p}{\frac{J}{kg°C}}$	Thermal conductivity $\frac{\lambda}{\frac{W}{m°C}}$	Melting point $\frac{t_s}{°C}$	Latent heat of fusion $\frac{s}{\frac{kJ}{kg}}$	Bulk modulus, modulus of compression $\frac{K}{10^9\,Pa}$	Resistivity at +20 °C $\frac{\rho_R}{10^{-9}\,\Omega m}$	Temperature coefficient of resistivity $\frac{\alpha_R}{10^{-3}\,\frac{1}{°C}}$
Aluminium	2700	23	70	25	909	217	660	687	70	27,2	4,0
Brass	8400	20	90	35	394	116			59	50	1,8
Copper	8930	17	120	46	389	393	1083	209	120	17,2	3,9
Invar	8100	0,8	145		502	16,2	1450			100	2,0
Silver	10500	19	80	29	230	400	960			15,8	4,0
Steel (Iron)	7830	12	210	85	473	58			160		
Zinc	7130	26	43	16	385	113	419	112		59,5	4,2
Glass		3,2	60		833						
Ice (0 °C)	920	0,9			2090	2,2	0	333			