1 a) $\neg p \wedge q$

| p | ¬p | q | p∧q | ¬(p∧q) |
|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |

b) $p \vee q \Rightarrow p \wedge q$

| p | q | p∨q | p∧q | p∨q ⇒ p∧q |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

c) $p \vee (q \Rightarrow r)$

| p | q | r | q⇒r | p∨(q⇒r) |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

2.

a) $\neg p \Leftrightarrow p \downarrow p$

b) $p \wedge q \Leftrightarrow (p \downarrow p) \downarrow (q \downarrow q)$

c) $p \vee q \Leftrightarrow (p \downarrow q) \downarrow (p \downarrow q)$

3 ~~$\neg(\neg(p \Rightarrow \neg q) \vee \neg(q \Rightarrow \neg p))$~~

~~$= (p \Rightarrow \neg q) \wedge (q \Rightarrow \neg p)$~~

~~$\Leftrightarrow (\neg p \vee \neg q) \wedge (\neg q \vee \neg p)$~~

$=$

3. $\neg(\neg(p \Rightarrow \neg q) \vee \neg(q \Rightarrow \neg p))$

$= \neg(\neg(p \Rightarrow \neg q) \vee \neg(\neg q \vee \neg p))$

$= \neg(\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg p))$

$= \neg(\neg(\neg p \vee \neg q) \vee \neg(\neg p \vee \neg q))$

$= \neg(\neg(\neg p \vee \neg q))$

$= \neg p \vee \neg q$

| p | q | ¬p | ¬q | ¬p∨¬q |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |

⇒ False

4. M : Marry
R : Regret

$(M \wedge R) \vee (\neg M \wedge R)$

| M | R | M∧R | ¬M∧R | ↓ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | (1) |

True, but also not true

5a) F: Fuel available
G: Go to store
C: Buy cookies

$$(F \Rightarrow G) \wedge (G \Rightarrow C) \wedge F \Rightarrow C$$

| F | G | C | F ⇒ G | G ⇒ C | F ⇒ C | — |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

⇒ True

b) S: Study hard
R: Get rich
G: Good grade

$$(S \vee R) \wedge G \Rightarrow \neg S \wedge R$$

| S | R | G | S ∨ R | ¬S ∧ R | — |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 1 |

⇒ False

6a) $\forall x \in \mathbb{Z}^+ : (1+2x)^2 > 1+2x$
False. $x = 1$

b) $\forall x \in \mathbb{R} : (1+x)^2 > 1+2x$
False: $x = -0.5$

c) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} : x+y = 2x-y$
True: $x = y$