**Statistical mathematics** Assignment 3 (Thursday 27/1)

Group 1 (10–12), Group 2 (12–14), Group 3 (14–16)

---

**1.** If $A \subseteq B$ and $B^c \neq \emptyset$, is $P(A)$ larger or smaller than $P(A|B)$? Give some reasoning for your answer.

**2.** Suppose that births are equally likely to be on any day. Let us also agree that there are 365 days in a year.

(a) What is the probability that somebody chosen at random has a birthday on the first day of a month?
(b) How does this probability change conditional on the knowledge that the person's birthday is in March?
(c) In February?

**3.** Suppose that we know the following percentages concerning the adult population of some country:

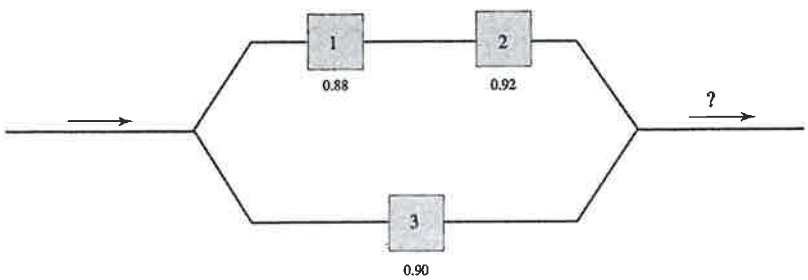
- The proportion of people who are both overweight and suffer hypertension is 10%.
- The proportion of people who are not overweight but suffer hypertension is 18%.
- The proportion of people who are overweight but do not suffer hypertension is 7%.
- The proportion of people who are neither overweight nor suffer hypertension is 65%.

An adult is randomly selected from this population.

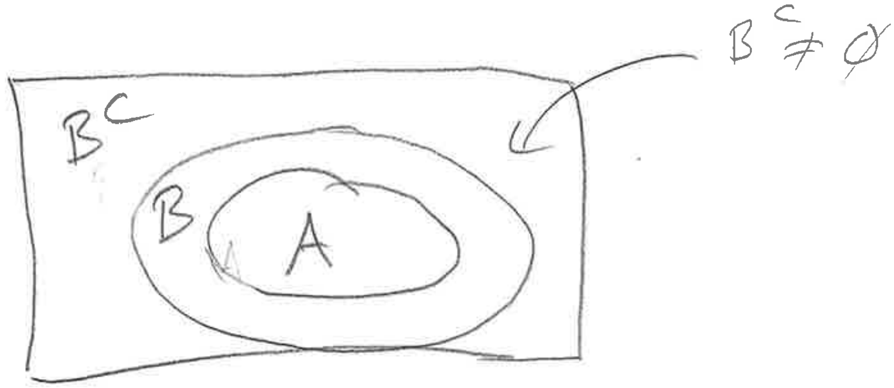
(a) Find the probability that the person selected suffers hypertension given that he is overweight.
(b) Find the probability that the selected person suffers hypertension given that he is not overweight.
(c) Compare the two probabilities just found to give an answer to the question as to whether overweight people tend to suffer from hypertension

**4.** Assume that $A$ and $B$ are such events that $P(A) = 0.4$ and $P(B) = 0.3$. We also know that $P(A^c \cap B^c) = 0.42$. Are the events $A$ and $B$ independent?

**5.** Let us consider the situation depicted in the following figure. The switches operate independently of one another. Switch 1 allows a message to go through with probability of 0.88, switch 2 allows a message to go through with probability of 0.92 and switch 3 allows a message to go through with a probability of 0.90. What is the probability that a message will find it's way through the network?



$$P(A|B) = \frac{P(A \cap B)}{P(B)} \overset{(A \subseteq B)}{=} \frac{P(A)}{P(B)}$$ ①




Because $B^c \neq \emptyset$, $P(B^c) > 0$ AND

$P(B) = 1 - P(B^c) < 1.$

This means that

$$P(A) < \frac{P(A)}{P(B)}$$

(2) a) $|S| = 365$ (DAYS IN A YEAR)

$|A| = 12$ (1st DAYS OF MONTHS)

$$P(A) = \frac{12}{365} = 0.0329$$

b) $|S'_h| = 31$ (NUMBER OF DAYS IN MARCH)

$|B| = 1$ (NUMBER OF 1st DAYS IN MARCH)

$$P(B) = \frac{1}{31} = 0.0323$$

ALSO AS CONDITIONAL PROBABILITY

C = "BIRTHDAY IS IN MARCH"

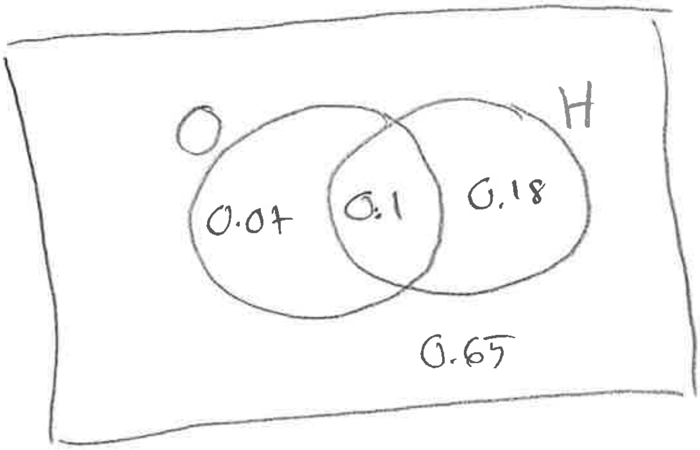
$$P(C) = \frac{31}{365}$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{1/365}{31/365} = 1/31$$

c) $1/28 = 0.0357$ (FEBRUARY)

③ O = "OVERWEIGHT"

H = "HYBERTENSION"




$P(O) = 0.1 + 0.07$
$= 0.17$

$P(O^c) = 1 - P(O)$
$= 0.83$

a) $P(H|O) = \frac{P(H \cap O)}{P(O)} = \frac{0.1}{0.17} = 0.588$

b) $P(H|O^c) = \frac{P(H \cap O^c)}{P(O^c)} = \frac{0.18}{0.83} = 0.217$

c) SUFFERING HYPERTENSION IS MORE COMMON AMONG OVERWEIGHTED PERSONS

(4) $A^c \cap B^c = (A \cup B)^c$

$$P(A \cup B) = 1 - P((A \cup B)^c) = 1 - 0.42$$
$$= 0.58$$

WE HAVE

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.4 + 0.3 - 0.58 = 0.12$$

$$P(A) \cdot P(B) = 0.4 \times 0.3 = 0.12$$

$\Rightarrow$ A AND B ARE INDEPENDENT

⑤ Let us denote

A = "MESSAGE GOES UPPER ROUTE"

B = "MESSAGE GOES LOWER ROUTE"

THE OPERATION OF SWITCHES IS INDEPENDENT

$P(A) = 0.88 \times 0.92 = 0.8096$

$P(B) = 0.9$

A AND B ARE INDEPENDENT, BUT NOT DISJOINT

$A \cup B$ MEANS THAT THE MESSAGE GOES

PROBLEM WE DO NOT KNOW $A \cap B$ ($\neq \emptyset$)

$$(A \cup B)^c = A^c \cap B^c$$

BECAUSE A AND B ARE INDEPENDENT, $A^c$ AND $B^c$ ARE INDEPENDENT (LECTURES)

$$P(A^c \cap B^c) = P(A^c)P(B^c) = (1 - P(A))(1 - P(B))$$
$$= (1 - 0.8096)(1 - 0.9) = 0.01904$$

$P(A \cup B) = 1 - 0.01904 = \underline{0.98096}$