Statistical mathematics Assignment 4 (Thursday 3/2)

Online Group 1 (10–12), Group 2 (12–14), Group 3 (14–16)

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1. Identify the range (= set of possible values) for each random variable.

(a) The number of heads in two tosses of a coin.
(b) The number of coins that match when three coins are tossed at once.
(c) A tennis match is divided up into sets. Typically, in men's tennis you have to get three sets to win. For women it's two. Consider these separately.
(d) The number of hearts in a five-card hand drawn from a deck of 52 cards that contains 13 hearts in all.
(e) The total number of goals in a soccer match

2. A welfare organization in a town organizes a lottery each month. One thousand lottery tickets are sold for 1 EUR each. Each has an equal chance of winning. First prize is 300 EUR, second prize is 200 EUR, and third prize is 100 EUR. Let X denote the **net gain** from the purchase of one ticket.

(a) Construct the probability mass function of X. Note that a ticket may not win.
(b) Find the probability of winning any money in the purchase of one ticket.

3. Suppose that a pair of dices is "loaded" in that way that the probability of getting 6 is twice as high than other numbers, meaning that the probability of 6 is $\frac{2}{7}$ and the probability for the other numbers $1, 2, 3, 4, 5$ is $\frac{1}{7}$.
Let X denote the sum of dices.

(a) What is the range R_X of X?
(b) Construct the mass function P_X for X for these loaded dices.

4. Determine whether or not the following tables are valid probability distributions of some discrete random variable. Explain.

(a)

x	0	1	2	3	4
$P(x)$	−0.25	0.5	0.35	0.1	0.3

(b)

x	home	draw	away
$P(x)$	0.325	0.406	0.164

(c)

x	25	26	27	28	29
$P(x)$	0.13	0.27	0.28	0.18	0.14

5. Prove that the geometric distribution satisfies

$$\sum_{k\geq 0} P(X=k) = 1.$$

You may need this geometric series formula ($r \neq 1$)

$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}$$

1. (a) $R_X = \{0, 1, 2\}$

$S = \{(T,T), (H,T), (T,H), (H,H)\}$

(b) $S = \{(T,T,T), (T,T,H), (T,H,T), (T,H,H),$
$(H,T,T), (H,T,H), (H,H,T), (H,H,H)\}$

ALWAYS AT LEAST TWO SAME! $R_X = \{2, 3\}$

(c) $S_{MEN} = \{3-0, 3-1, 3-2, 0-3, 1-3, 2-3\}$

$R_X = \{3, 4, 5\}$

$S_{WOMEN} = \{2-0, 2-1, 0-2, 1-2\}$

$R_X = \{2, 3\}$

(d) $R_X = \{0, 1, 2, 3, 4, 5\}$

(e) RECORD IS 149–0. THE TEAM MADE AS MANY OWN GOALS AS POSSIBLE (PROTEST)

$R_X = \{0, 1, 2, 3, \ldots, ?\}$

(2) $R_X = \{300-1, 200-1, 100-1, 0-1\}$

$= \{299, 199, 99, -1\}$

(a) $P(X=299) = 1/1000$

$P(X=199) = 1/1000$

$P(X=99) = 1/1000$

$P(X=-1) = 997/1000$

b) $3/1000$

③ Let us consider the elements of the sample space

	1	2	3	4	5	6
1	$1/49$	$1/49$	$1/49$	$1/49$	$1/49$	$2/49$
2	$1/49$	$1/49$	$1/49$	$1/49$	$1/49$	$2/49$
3	$1/49$	$1/49$	$1/49$	$1/49$	$1/49$	$2/49$
4	$1/49$	$1/49$	$1/49$	$1/49$	$1/49$	$2/49$
5	$1/49$	$1/49$	$1/49$	$1/49$	$1/49$	$2/49$
6	$2/49$	$2/49$	$2/49$	$2/49$	$2/49$	$4/49$

(a) $R_X = \{2,3,4,5,6,7,8,9,10,11,12\}$

(b) $P(X=2) = 1/49$, $P(X=3) = 2/49$, $P(X=4) = 3/49$

$P(X=5) = 4/49$, $P(X=6) = 5/49$

$P(X=7) = \frac{1}{49}(2+1+1+1+1+2) = 8/49 \approx 0.163$

$P(X=8) = \frac{1}{49}(2+1+1+1+2) = 7/49 = 1/7$

$P(X=9) = \frac{1}{49}(2+1+1+2) = 6/49$

$P(X=10) = \frac{1}{49}(2+1+2) = 5/49$

$P(X=11) = \frac{1}{49}(2+2) = \frac{4}{49}$

$P(X=12) = \frac{4}{49}$

(4) a) NO. $P_X(0)$ IS NEGATIVE

b) NO. SUM IS $0.895 < 1$

(c) YES: $0 \leq P_X(x) \leq 1$ for all $x \in R_X$

and $\sum_{x \in R_X} P_X(x) = 1$

(5) $P(X=k) = (1-p)^{k-1} \cdot p \quad \boxed{k \geq 1}$

Let us denote $q = 1-p$

$$\sum_{k \geq 1} P(X=k) = \sum_{k \geq 1} (1-p)^{k-1} \cdot p = \sum_{k \geq 1} q^{k-1} \cdot p$$

$$= p \sum_{k \geq 1} q^{k-1}$$

$$= p\left(q^0 + q^1 + q^2 + q^3 + \dots\right)$$

$$= p \cdot \frac{1}{1-q}$$

$$= p \cdot \frac{1}{p} = \underline{\underline{1}}$$