Task 1

List of symbols can be found on page 5.

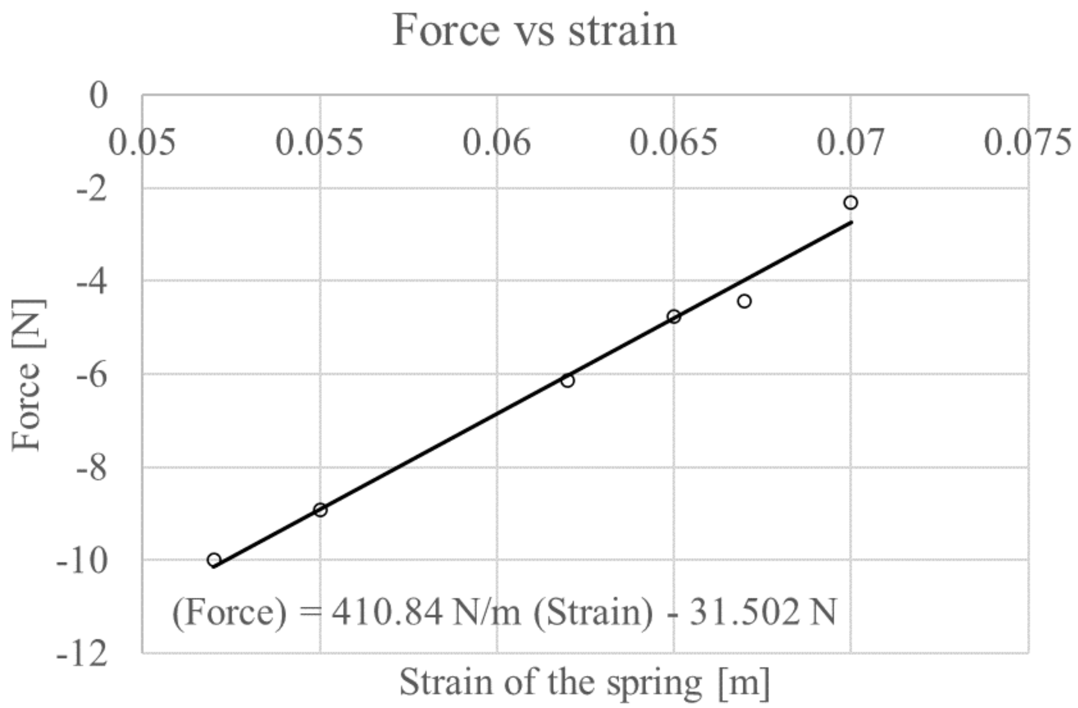
a) $\theta = 90° \pm n180°$

$\sin\theta = \pm 1$

$$\boldsymbol{v}_{\mathbf{max}} = \omega A|\sin\theta| = \omega A|\pm 1| = \sqrt{\frac{\boldsymbol{k}}{\boldsymbol{m}}}\boldsymbol{A}$$

b) $\boldsymbol{T} = \frac{1}{f} = \frac{2\pi}{\omega} = \boldsymbol{2\pi}\sqrt{\frac{\boldsymbol{m}}{\boldsymbol{k}}}$

Task 2



Spring constant:

$$k = 410.84\,\frac{\mathrm{N}}{\mathrm{m}}$$

Length of the unstrained spring:

$$\frac{31.502\ \mathrm{N}}{410.84\ \mathrm{N/m}} = 7.6677 \ldots \mathrm{cm}$$

Task 3

List of symbols can be found on page 5.

$x = A\sin\theta = \frac{A}{2}$

$\sin\theta = \frac{1}{2}$

$(\sin\theta)^2 + (\cos\theta)^2 = 1$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\omega = 2\pi f$

(1) $|\boldsymbol{v}| = |\omega A\cos\theta| = |2\pi f A\cos\theta| = \left|\frac{\sqrt{3}}{2}2\pi f A\right| = \boldsymbol{\sqrt{3}\pi f A}$
(2) $|\boldsymbol{a}| = |-\omega^2 A\sin\theta| = \left|\frac{(2\pi f)^2 A}{2}\right| = \boldsymbol{2A(\pi f)^2}$
(3) $\boldsymbol{\frac{1}{2}mv^2} = \frac{1}{2}m\left(\sqrt{3}\pi f A\right)^2 = \boldsymbol{\frac{3}{2}m(\pi f A)^2}$

Task 4

Mass of the Benji jumper	m	[kg]
Height from the water that the jumper reaches	H	[m]
Height of the bridge from the water	H_{bridge}	[m]
Length of the unstretched rope	$l_{\text{rope,0}}$	[m]
Length of the stretched rope when the jumper is at H	l_{rope}	[m]
Length that the rope has stretched when the jumper is at H	Δl_{rope}	[m]
Spring constant of the rope	k	[N/m]
Gravitational acceleration	g	[m/s^2]
Acceleration of the jumper at H	a	[m/s^2]

Formulas:

Energy stored in the stretched rope when height of the jumper is H	$\frac{1}{2}k\Delta l_{\text{rope}}^2$
Gravitational potential energy lost by the jumper when moving from H_{bridge} to H	$mg(H_{\text{bridge}} - H)$

$$\frac{1}{2}k\Delta l_{\text{rope}}^2 = mg(H_{\text{bridge}} - H)$$

$$\Delta l_{\text{rope}} = l_{\text{rope}} - l_{\text{rope,0}}$$

$$l_{\text{rope}} = H_{\text{bridge}} - H$$

$$k\Delta l_{\text{rope}} - mg = ma$$

Solution

Spring constant:

$$\Delta l_{\text{rope}} = H_{\text{bridge}} - H - l_{\text{rope,0}}$$

$$\boldsymbol{k = \frac{2mg(H_{\text{bridge}} - H)}{\Delta l_{\text{rope}}^2} = \frac{2mg(H_{\text{bridge}} - H)}{(H_{\text{bridge}} - H - l_{\text{rope,0}})^2}}$$

Acceleration as the jumper starts to rise:

$$k\Delta l_{\text{rope}} - mg = ma$$

$$\boldsymbol{a = \frac{k(H_{\text{bridge}} - H - l_{\text{rope,0}})}{m} - g}$$

Mass of the jumper if $H = 0$:

$$\frac{1}{2}k(H_{\text{bridge}} - H - l_{\text{rope,0}})^2 = mg(H_{\text{bridge}} - H)$$

$$\frac{1}{2}k(H_{\text{bridge}} - l_{\text{rope,0}})^2 = mgH_{\text{bridge}}$$

$$\boldsymbol{m = \frac{k(H_{\text{bridge}} - l_{\text{rope,0}})^2}{2gH_{\text{bridge}}}}$$

Task 5

Effective spring constant of the system of two springs and a mass	k	[N/m]
Spring constants of the two springs in the system	k_1,k_2	[N/m]
Steady acceleration of the car	a_1	[m/s^2]
Steady acceleration of the car in the sudden braking	a_2	[m/s^2]
Velocity during the sudden braking	v_2	[m/s]
Position of the object	x	[m]
Mass of the object	m	[kg]
Initial velocity of the driver's head towards the steering wheel in the sudden braking	v_0	[m/s]
Distance between the steering wheel and the driver's head before the sudden braking	D	[m]
Time at which the sudden breaking starts	t_0	[s]
Time at which the drivers head hits the steering wheel contact of driver's head to the steering wheel	t_1	[s]
Time	t	[s]
Time it takes that the object reaches its peripheral position from the opposite peripheral position	T_{p}	[s]

Formulas:

$$k = k_1 + k_2$$

$$v = \int_{t_0}^{t} a_2 \, dt = a_2 t + v_0$$

$$D = \int_{t_0}^{t_1} v dt = \frac{1}{2} a_2 t^2 + v_0 t$$

$$T_{\mathrm{p}} = \frac{T}{2}$$

When the car steadily accelerates:

$$F = m a_1 = kx$$

$$\boldsymbol{x} = \frac{m a_1}{k} = \frac{\boldsymbol{m a_1}}{\boldsymbol{k_1 + k_2}}$$

When the car decelerates at the constant rate a_2:

$$D = \frac{1}{2} a_2 t^2 + v_0 t$$

$$v_0 = 0$$

$$D = \frac{1}{2} a_2 t^2$$

$$\boldsymbol{t} = \sqrt{\frac{\boldsymbol{2D}}{\boldsymbol{a_2}}}$$

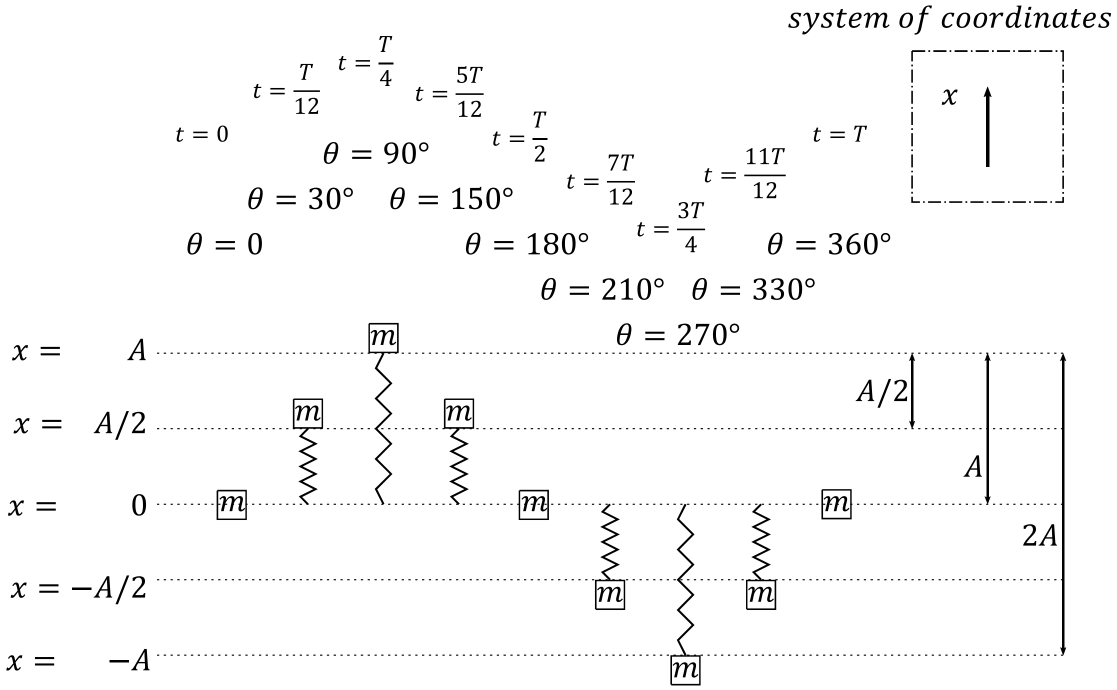
The object moves half a period of oscillation:

$$\boldsymbol{T_{\mathrm{p}}} = \frac{T}{2} = \pi \sqrt{\frac{m}{k}} = \boldsymbol{\pi} \sqrt{\frac{\boldsymbol{m}}{\boldsymbol{k_1 + k_2}}}$$

Assignment 5

Harmonic oscillator theory review sheet:

Point-like mass	m	[kg]
Spring constant	k	[N/m]
Displacement of the point-like mass from the unstrained position	x	[m]
Amplitude of oscillation	A	[m]
Period of oscillation	T	[s]
Force acting on the point-like mass	F	[N]
Acceleration of the point-like mass	a	[m/s^2]
Position of the point-like mass expressed as an angle	θ	[degrees]
Angular velocity of the point-like mass	ω	[degrees/s]
Velocity of the point-like mass	v	[m/s]
Time elapsed	t	[s]
Frequency of the oscillation	f	[1/s]



$F = ma = -kx$

$x(t = 0) = 0$ $\qquad$ $x(\theta = 90°) = A$

see solution of the differential equation

$$x = A \sin\theta$$

$$v = \frac{dx}{dt} = \omega A \cos\theta$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \sin\theta$$

$$\theta = \omega t \quad \omega = \sqrt{\frac{k}{m}} \quad \omega = 2\pi f \quad T = \frac{1}{f}$$

Solution of the differential equation

Constants in the general solution of the differential equation C_1, C_2 *[m]*

Differential equation to be solved:

$$\boldsymbol{ma(t) = -kx(t)}$$

Initial conditions:

$$\boldsymbol{x(t = 0) = 0}$$

$$\boldsymbol{x(\theta = 90°) = A}$$

Rearrange:

$$ma + kx = 0$$

$$\boldsymbol{a + \frac{k}{m}x = 0}$$

Ordinary homogeneous differential equation with constant coefficients

Characteristic polynomial

$$r^2 + \frac{k}{m} = 0$$

$$r = \sqrt{-\frac{k}{m}} = \pm\sqrt{\frac{k}{m}}i$$

Form of the solution is

$$x = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right)$$

Initial condition $x(t = 0) = 0$*:*

$$x(t = 0) = C_1 \cos(0) + C_2 \sin(0) = C_1 = 0$$

Initial condition $x(\theta = 90°) = A$*:*

$$x(\theta = 90°) = C_2 \sin 90° = C_2 = A$$

Final solution:

$$x = A \sin\left(\sqrt{\frac{k}{m}}t\right)$$

$$where \sqrt{\frac{k}{m}} = \omega$$