Task 1

Peripheral (or tangential) velocity of the car	v	$[\mathrm{m/s}]$
Time period that one round takes	T	$[s]$
Frequency of the periodic movement of the car	f	$[1/\mathrm{s}]$
Angular velocity	ω	$[\mathrm{rad}/s]$
Radius of the circle	r	$[m]$

Equations:

$\omega = 2\pi f$

$f = \frac{1}{T}$

$\omega = \frac{v}{r}$

Solution:

A)

$\omega = 2\pi f = \frac{2\pi}{T} = \frac{v}{r}$

solve for r

$r = \frac{vT}{2\pi} = \frac{(50\frac{1\ \mathrm{m}}{3.6\ \mathrm{s}})(24\ \mathrm{s})}{2\pi} = 53.0516476972\ldots\mathrm{m} \approx 53.1\ \mathrm{m}$

B)

$\omega = \frac{2\pi}{T} = \frac{2\pi}{(24\ \mathrm{s})} = 0.26179938779\frac{\mathrm{rad}}{\mathrm{s}} \approx 0.262\frac{\mathrm{rad}}{\mathrm{s}}$

Task 2

Frequency of the jet engine 1	f_1	[1/s]
Frequency of the jet engine 2	f_2	[1/s]
Beat frequency	f_B	[1/s]
Average frequency	f_A	[1/s]

Equations:

$$f_\mathrm{B} = |f_2 - f_1|$$

$$f_\mathrm{A} = \frac{f_1 + f_2}{2}$$

Solution:

Assume f_2 is higher than f_1, then:

$$\begin{cases} -f_1 + f_2 = f_\mathrm{B} \\ 0.5f_1 + 0.5f_2 = f_\mathrm{A} \end{cases}$$

Solution:

$$f_1 = \frac{\begin{vmatrix} f_\mathrm{B} & 1 \\ f_\mathrm{A} & 0.5 \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ 0.5 & 0.5 \end{vmatrix}} = f_\mathrm{A} - 0.5f_\mathrm{B} = 4100\ \mathrm{Hz} - 0.5(0.500\ \mathrm{Hz}) = 4099.75\ \mathrm{Hz} \approx 4099.8\ \mathrm{Hz}$$

$$f_2 = \frac{\begin{vmatrix} -1 & f_\mathrm{B} \\ 0.5 & f_\mathrm{A} \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ 0.5 & 0.5 \end{vmatrix}} = f_\mathrm{A} + 0.5f_\mathrm{B} = 4100\ \mathrm{Hz} + 0.5(0.500\ \mathrm{Hz}) = 4100.25\ \mathrm{Hz} \approx 4100.3\ \mathrm{Hz}$$

Task 3

Mass connected to the spring	m	[kg]
Spring constant	k	[N/m]
Maximum velocity of the mass	v_{max}	[m/s]
Velocity of the mass	v	[m/s]
Amplitude of the mass vibrating	A	[m]
Spring (potential) energy	E_{s}	[J]
Kinetic energy of the mass	E_{kin}	[J]

Equations:

$$E_{\text{s}} = \frac{1}{2}kx^2$$

$$E_{\text{kin}} = \frac{1}{2}mv^2$$

$$x = A\sin\theta$$

$$v = \omega A\cos\theta$$

$$v_{\text{max}} = \omega A$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

Solution:

A) $A = \frac{v_{\text{max}}}{\omega} = \sqrt{\frac{mv_{\text{max}}^2}{k}} = \sqrt{\frac{(11\text{ kg})\left(10\frac{\text{m}}{\text{s}}\right)^2}{\left(43\frac{\text{N}}{\text{m}}\right)}} = 5.057805388\text{ m} \approx 5.06\text{ m}$

B) $E_{\text{s}} = E_{\text{kin}}$ — substitute expressions

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ — substitute expressions

$k(A\sin\theta)^2 = m(\omega A\cos\theta)^2$ — divide both sides by $k(A\cos\theta)^2$

$\left(\frac{\sin\theta}{\cos\theta}\right)^2 = \frac{m\omega^2}{k}$ — substitute $\omega = \sqrt{\frac{k}{m}}$ and $\frac{\sin\theta}{\cos\theta} = \tan\theta$

$(\tan\theta)^2 = 1$ — take square root

$\tan\theta = \pm 1$

$\theta = \pm 45° \pm N \cdot 180°$

$\sin\theta = \pm 1/\sqrt{2}$

$$x = A\sin\theta = \pm\sqrt{\frac{mv_{\text{max}}^2}{2k}} = \pm\sqrt{\frac{(11\text{ kg})\left(10\frac{\text{m}}{\text{s}}\right)^2}{2\left(43\frac{\text{N}}{\text{m}}\right)}} = \pm 3.57640848\ldots\text{ m} \approx \pm 3.58\text{ m}$$

Task 4

Mass connected to the spring	m	[kg]
Spring constant	k	[N/m]
Distance that the spring is stretched from unstrained position	Δx	[m]
Potential energy of the mass after descending	E_{pot}	[J]
Spring (potential) energy	E_{s}	[J]
Gravitational acceleration	g	[m/s^2]

Formulas:

$$E_{\mathrm{pot}} = mg\Delta x$$

$$E_{\mathrm{s}} = \frac{1}{2}kx^2$$

$$mg = k\Delta x$$

Solution:

A) $\Delta x = \frac{mg}{k} = \frac{(0.500\ \mathrm{kg})\left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)}{\left(40.0\frac{\mathrm{N}}{\mathrm{m}}\right)} = 0.122625\ \mathrm{m} \approx 0.12\ \mathrm{m}$

B) $E_{\mathrm{pot}} = mg\Delta x = \frac{(mg)^2}{k} = \frac{\left[(0.500\ \mathrm{kg})\left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)\right]^2}{\left(40.0\frac{\mathrm{N}}{\mathrm{m}}\right)} = 0.601475625\ \mathrm{J} \approx 0.60\ \mathrm{J}$

C) $E_{\mathrm{s}} = \frac{1}{2}k\Delta x^2 = \frac{1}{2}\frac{(mg)^2}{k} = 0.5\ E_{\mathrm{pot}} = 0.3007378125\ \mathrm{J} \approx 0.30\ \mathrm{J}$

$E_{\mathrm{s}} = \frac{1}{2}E_{\mathrm{pot}}$

Task 5

Mass connected to the spring	m	[kg]
Spring constant	k	[N/m]
Initial displacement	X	[m]
Final displacement	x	[m]
Total distance that the mass slides as it slides back and forth along the surface about the unstretched position	d	[m]
Friction coefficient between the mass and the surface	μ	[−]
Energy dissipated into heat due to friction	E_{f}	[J]
Initial spring potential energy	E_{SI}	[J]
Final spring potential energy	E_{SF}	[J]

Formulas:

$$E_{\mathrm{SI}} - E_{\mathrm{SF}} = E_{\mathrm{f}}$$

$$E_{\mathrm{f}} = mg\mu d$$

$$E_{\mathrm{SI}} = \frac{1}{2}kX^2$$

$$E_{\mathrm{SF}} = \frac{1}{2}kx^2$$

$$mg\mu = kx$$

Solution:

$$x = \frac{mg\mu}{k}$$

$$E_{\mathrm{SI}} - E_{\mathrm{SF}} = E_{\mathrm{f}}$$ substitute expressions

$$\frac{1}{2}(kX^2 - kx^2) = mg\mu d$$ substitute expressions

$$\frac{1}{2}\left[kX^2 - k\left(\frac{mg\mu}{k}\right)^2\right] = mg\mu d$$ rearrange

$$\frac{(mg)^2}{2k}\mu^2 + mgd\mu - \frac{1}{2}kX^2 = 0$$ use quadratic formula

$$\mu = \frac{-mgd \pm \sqrt{(mgd)^2 + (mgX)^2}}{\frac{(mg)^2}{k}}$$

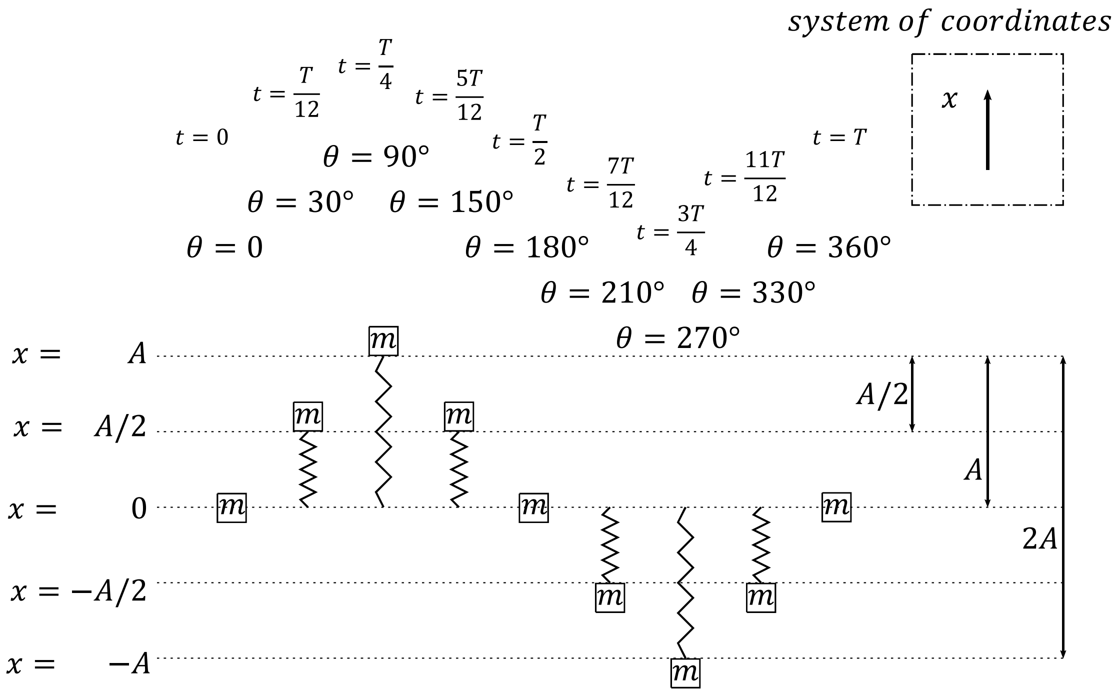
$$= \frac{k\left(-d \pm \sqrt{d^2 + X^2}\right)}{mg} = \frac{\left(43\,\frac{\mathrm{N}}{\mathrm{m}}\right)\left[-(4\ \mathrm{m}) \pm \sqrt{(4\ \mathrm{m})^2 + (2\ \mathrm{m})^2}\right]}{(11\ \mathrm{kg})\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)} = 0.1881368368 \approx 0.188$$

$$x = \frac{mg\mu}{k} = -d \pm \sqrt{d^2 + X^2} = -(4\ \mathrm{m}) \pm \sqrt{(4\ \mathrm{m})^2 + (2\ \mathrm{m})^2} = 0.47213\ \mathrm{m} \approx 0.47\ \mathrm{m}$$

Assignment 6

Harmonic oscillator theory review sheet:

Point-like mass	m	[kg]
Spring constant	k	[N/m]
Displacement of the point-like mass from the unstrained position	x	[m]
Amplitude of oscillation	A	[m]
Period of oscillation	T	[s]
Force acting on the point-like mass	F	[N]
Acceleration of the point-like mass	a	[m/s^2]
Position of the point-like mass expressed as an angle	θ	[degrees]
Angular velocity of the point-like mass	ω	[degrees/s]
Velocity of the point-like mass	v	[m/s]
Time elapsed	t	[s]
Frequency of the oscillation	f	[1/s]



$F = ma = -kx$

$x(t = 0) = 0$ $\qquad$ $x(\theta = 90°) = A$

see solution of the differential equation

$$x = A \sin \theta$$

$$v = \frac{dx}{dt} = \omega A \cos \theta$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \sin \theta$$

$$\theta = \omega t \quad \omega = \sqrt{\frac{k}{m}} \quad \omega = 2\pi f \quad T = \frac{1}{f}$$

Solution of the differential equation

Constants in the general solution of the differential equation C_1, C_2 *[m]*

Differential equation to be solved:

$$\boldsymbol{ma(t) = -kx(t)}$$

Initial conditions:

$$\boldsymbol{x(t = 0) = 0}$$

$$\boldsymbol{x(\theta = 90°) = A}$$

Rearrange:

$$ma + kx = 0$$

$$\boldsymbol{a + \frac{k}{m}x = 0}$$

Ordinary homogeneous differential equation with constant coefficients

Characteristic polynomial

$$r^2 + \frac{k}{m} = 0$$

$$r = \sqrt{-\frac{k}{m}} = \pm\sqrt{\frac{k}{m}}i$$

Form of the solution is

$$x = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right)$$

Initial condition $x(t = 0) = 0$*:*

$$x(t = 0) = C_1 \cos(0) + C_2 \sin(0) = C_1 = 0$$

Initial condition $x(\theta = 90°) = A$*:*

$$x(\theta = 90°) = C_2 \sin 90° = C_2 = A$$

Final solution:

$$x = A \sin\left(\sqrt{\frac{k}{m}}t\right)$$

$$\textit{where } \sqrt{\frac{k}{m}} = \omega$$