LES10A200 – Engineering Physics / Home Assignment 8

Task 1

Given values:

Air temperature = 15 °C

Relative air humidity = 60 %

To find relative humidity at 30 °C?

Also, how many percentages of the volume of the gas expanded due to heating?

Assumptions: 1 atm pressure and that ideal gas law applies

Solution 1:

Relative humidity, R.H. = $\frac{Partial\ pressure\ of\ water\ vapor\ at\ T}{full\ vapor\ pressure\ of\ water\ at\ T} * 100\%$

For given conditions at 15 °C, the relative humidity was given as 60 %,

Therefore, 0.6 = $\frac{Partial\ pressure\ of\ water\ vapor\ at\ T}{full\ vapor\ pressure\ of\ water\ at\ T}$.

From the table given in Lecture notes, the full vapor pressure of water at 15 °C = 1705.

Therefore, the partial pressure of water vapor at 15 C = 0.6 * 1705 Pa = 1023 Pa.

Now, we use this partial pressure of water vapor to find the relative humidity at 30 °C:

R.H. = $\frac{Partial\ pressure\ of\ water\ vapor\ at\ T}{full\ vapor\ pressure\ of\ water\ at\ T} * 100\% = \frac{1023}{4243} * 100\% = 24.11\%$

Since, the gas follows the ideal gas law: PV = nRT.

So, PV/T = constant. Also, P is assumed to be 1 atm.

Therefore, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$,

Expansion due to heating: From T_1=15 °C to T_2=30 °C

The expansion in the volume is $V_2 = \frac{T_2}{T_1} V_1 = \frac{30+273.15}{15+273.15} V_1 = 1.052 V_1$

The increase in the percentage of the volume due to heating is 5.2 %

Task 2

Given values:

Refrigeration coefficient of performance = 2.5

Fridge's compressor runs with 340 W power.

To find:

a) How much heat per second does it remove from the refrigerator?

b) If same unit is used as a heating pump, what would be its coefficient of performance?

Solution 2:

For refrigeration, we have a coefficient of performance as:

$$K_R = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

For a), we have, K_R = 2.5, therefore, $Q_L = K_R * W = 2.5 * 340 = 850\ W$.

So, this unit is for power, and it is Joules/s. So, we have 850 J of heat removed per second.

For b) if same unit is used as a heating pump, then we have the formula,

$$K_P = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L}$$

W = Q_H-Q_L

So, Q_H = W+Q_L = 340 + 850 = 1190 W.

So, coefficient of performance, $K_P = \frac{Q_H}{W} = \frac{1190}{340} = 3.5$

Task 3

Given values: At standard temperature and pressure STP: Temperature = 0 ℃ and 100 kPa

Given values: At normal temperature and pressure NTP: Temperature = 20 ℃ and 101.325 kPa

Assuming, air is following the ideal gas law: PV=nRT

One mole of an ideal gas has a volume of 22.41 litres in STP conditions.

Relative air humidity at STP= 100 %

To find relative humidity of air at the NTP?

Also, how many grams of water vapor does 1 m^3 of this air contain in NTP conditions?

Solution 3:

For a) Relative humidity, R.H. = $\frac{Partial\ pressure\ of\ water\ vapor\ at\ T}{full\ vapor\ pressure\ of\ water\ at\ T} * 100\%$

For STP conditions case, the relative humidity was given as 100 %.

100% = $\frac{Partial\ pressure\ of\ water\ vapor\ at\ T}{full\ vapor\ pressure\ of\ water\ at\ T} * 100\%$.

Therefore, the partial pressure of water vapor at 0 ℃ is same as the full vapor pressure of water at

0 ℃ = 610.5 Pa (from the table given in Lecture notes)

Now, we use this partial pressure of water vapor to find the relative humidity at NTP conditions.

R.H. (NTP) = $\frac{Partial\ pressure\ of\ water\ vapor\ at\ T}{full\ vapor\ pressure\ of\ water\ at\ T} * 100\% = \frac{610.5}{2338} = 0.2611 = 26.11\%$

For b) since it is following the ideal gas law, PV = nRT, we use the ratio p_i/n_i = constant, for the components.

Therefore, $p_{NTP}/n_{air,NTP} = p_{H_2O,NTP}/n_{H_2O}$.

Based on this correlation, we can get, $n_{H_2O} = \frac{p_{H_2O,NTP}}{p_{NTP}} * n_{air,NTP}$.

Here, first to calculate, $n_{air,NTP}$ using the relation PV=nRT.

Therefore, $n_{air,NTP}$ = PV/RT = 101325 * 1 / (8.314*293.15) = 41.5735. (Based on the volume as 1 cubic meter).

Using this value to get $n_{H_2O} = \frac{2338}{101325} * 41.5735$ = 0.959278 moles of H_2O at NTP.

For grams of water vapor, m = n_{H2O}*M = 0.959278 mol * 18.015 g/mol = 17.28 g.

Task 4

Given values:

1 m^3 volume of nitrogen at NTP conditions (101325 Pa and 20 °C) is heated to 100 °C in constant pressure.

To find: Comparison of the expansion of the gas when using the ideal gas law and Van der Waals equation by finding the percentages of the expansion of the gas? Also, to check how much percentage units is the difference?

Solution 4:

At initial conditions: T_0 = 20 °C $= 293.15\ K$, P_0 = 101325 Pa, V_0 = 1 m^3

At final conditions: T_f = 100 °C $= 373.15\ K$, P_f = 101325 Pa, V_f = ?

Using ideal gas law, PV = nRT

Number of moles at the initial condition: n = $\frac{PV}{RT} = \frac{101325*1}{8.314*293.15} = 41.57351007$

Using this value to get the expansion in the volume at the temperature 373.15 K:

$$V_f = \frac{nRT_f}{P_f} = \frac{41.57351007 * 8.314 * 373.15}{101325} = 1.272897834\ m^3$$

Therefore, the expansion in the volume in percentages is $\frac{V_f - V_o}{V_0} * 100\% = 27.2897834\%$

Using Van der Waals law, $\left[p + a\left(\frac{n}{V}\right)^2\right](V - nb) = nRT$

To convert the Van der Waals constants for the Nitrogen into specific units:

For constant a: 1.37 bar*dm^6 / mol^2 = 0.137 m^6 Pa/mol^2

For constant b: 0.0387 dm^3/mol = 0.0000387 m^3/mol

Number of moles at the initial condition, n = 41.60371 using mathematical solver.

Using this value to get the expansion in the volume at the temperature 373.15 K:

$$V_f = 1.273597\ m^3$$

Therefore, the expansion in the volume in percentages is $\frac{V_f - V_o}{V_0} * 100\% = 27.3597\%$

Difference in answers: 0.07 percentage units

Task 5

Given values:

Pressure of the car tire is 2.2 bar at -5 ℃. While driving, the temperature raises to 57 ℃.

To find: the pressure of the tire after the drive using the ideal gas law and Van der Waals equation? Also, to check how much percentage units is the difference?

Solution 4:

At initial conditions: T_0 = -5 ℃ $= 268.15\ K$, P_0 = 2.2 bar, V_0 = 10.6 litres = 0.0106 m^3

At final conditions: T_f = 57 ℃ $= 330.15\ K$, V_f = 0.0106 m^3, P_f = ?

Using ideal gas law, PV = nRT

Number of moles at the initial condition: n = $\frac{PV}{RT} = \frac{220000*0.0106}{8.314*268.15} = 1.046022$

Using this value to get the increase in the pressure of the tire after the drive at temperature 330.15 K:

$$P_f = \frac{nRT_f}{V_f} = \frac{1.046022 * 8.314 * 330.15}{0.0106} = 270867\ Pa$$

Using Van der Waals law, $\left[p + a\left(\frac{n}{V}\right)^2\right](V - nb) = nRT$

The partial pressures are obtained for each component of the air based on their compositions:

$p_{N2} = 0.781 * 2.2\ bar = 171820\ Pa,$

$p_{O2} = 0.21 * 2.2\ bar = 46200\ Pa,$

$p_{Ar} = 0.009 * 2.2\ bar = 1980\ Pa$

Note that the Van der Waals constants ‘a’ and ‘b’ are converted into the specific units as similar in Task 4 for all the three components of the gas.

Based on the partial pressures obtained above, using the Van der Waals law, the number of moles can be obtained for each component of the gas individually as below,

$$n_{N2} = 0.818844$$

$$n_{O2} = 0.219768$$

$$n_{Ar} = 0.009414$$

Based on the number of the moles obtained, the Van der Waals law can be used to find the final pressure values of the tire after the drive for each component of the gas at temperature 330.15 K as below,

$P_{N2,f} = 211220.7\ Pa,$

$P_{O2,f} = 56851.9\ Pa,$

$P_{Ar,f} = 2437.8\ Pa$

The total pressure of the tire after the drive is the sum of the individual component's pressure,

$P_{total,f} = P_{N2,f} + P_{O2,f} + P_{Ar,f}$=270510 Pa.

Difference in answers when using the Ideal gas law and the Van der Waals law:

270867 Pa – 270510 Pa = 357 Pa