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LES10A020 Engineering Physics

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# Engineering Physics

by

Assoc. Prof. Jukka Paatero

# Developing the Concepts of Thermodynamics

# Heat Quantity

- Heat Quantity needed to heat up substance was defined in the book as

$$Q = mc\Delta T$$

- However, the heat capacity $c$ can not always be considered constant, but a function of temperature $T$.
- Using a differential change in the quantities, we get

$$dQ = mc(T)dT$$

- Integrating  both sides, we get for temperature range $T_1 \ldots T_2$

$$Q = m\int_{T_1}^{T_2} c(T)dT$$

- When the heat capacity is assumed to be constant, this gives back the original equation.

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# Example 1: NaCl at Low $T$

- At low temperatures (<15K) we can approximate the heat capacity of NaCl with equation

$$c(T) = 3.33 \cdot 10^6 \frac{\text{J}}{\text{kg}\cdot\text{K}} \left(\frac{T}{321\text{K}}\right)^3$$

- Using this knowledge, how much heat is required to lift the temperature of 24g of NaCl from 5K to 15K?
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# Example 1: Solution

- Using the result from before, we have

$$Q = m \int_{T_1}^{T_2} c(T) dT$$

- Substituting the non-linear heat capacity, we get

$$Q = 0.024\ \mathrm{kg} \cdot \int_{T_1}^{T_2} 3.33 \cdot 10^6 \frac{\mathrm{J}}{\mathrm{kg \cdot K}} \left( \frac{T}{321\mathrm{K}} \right)^3 dT$$

$$= \left( 6.04 \cdot 10^{-4} \frac{\mathrm{J}}{\mathrm{K}^4} \right) T^4 \Bigg|_{5\mathrm{K}}^{15\mathrm{K}} = 0.302\ \mathrm{J}$$

- The value would be quite different, had we used the heat capacity for room temperature $880\ \mathrm{J/kg \cdot K}$.

# Conduction Revisited

- Let us look in more detail the Conduction chapter in the Dirks & Sharma: College Physics, Chapter 14.5 and the examples there.

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# Differential Conduction

- For the power of heat linear transfer, we can write

$$P = Q/t = \frac{kA(T_2 - T_1)}{d}$$

- Above $k$ is the thermal conductivity of the material in question.
- Using differential equations, the power of heat transfer can be rewritten as

$$P = \frac{dQ}{dt} = -kA\frac{dT}{dx}$$

- This equation can now be solved for non-linear situations via integration.
  - Typically, $k(x)$ can be non-linear in non-homogenous substances.
- Phase change calculations do not benefit from differentiation, as the melting and evaporation temperatures are constant values.

# Laws of Thermodynamics Revisited

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# First Rule of Thermodynamics

- In thermodynamics the change typically happens in the internal energy of the system, resulting in

$$\Delta U = W + Q$$

- In advanced thermodynamics, this is often used in the differential form, resulting in

$$dU = dW + dQ$$

- This allows solving non-linear situations via integration.

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# Revisiting Work

- Using vector notation, we can note work as

$$W = \bar{\mathbf{F}} \cdot \bar{\mathbf{s}}$$

- However, in differential form we typically write it as

$$dW = \bar{\mathbf{F}} \cdot d\bar{\mathbf{s}}$$

- Here the infinitesimal work done is considered as a function of infinitesimal change in displacement.
  - Further, this allows us to deal with non-linear force-vectors, as differentiation of the dot-product allows dealing with outcomes that are functions of displacement form
  - As we connect how displacement and time are related, by change variables we can also deal time-dependent force.

$$d\bar{\mathbf{s}}/dt = \bar{\mathbf{v}}\,(t)$$

# Second Law of Thermodynamics Revisited

- Change in Entropy for a reversible process is defined as: $$\Delta S = \left(\frac{Q}{T}\right)_{\text{rev}}$$
- For the Carnot heat engine, the ratio of heat quantity and temperature on cold and hot side is constant.
- Thus, we have: $$\frac{Q_c}{T_c} = \frac{Q_h}{T_h}$$
- When calculating for change in entropy, we get

$$\Delta S_{tot} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} = 0$$

- Overall, we can generalize the result: for reversible systems, change of entropy is zero.

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# Differential Form for Entropy

- We can also write the equation for entropy in differential form:

$$dS = \frac{dQ}{T}$$

- This form is particularly useful in detailed analysis of the internal energy and dealing with non-reversible processes.
- Also, non-linear temperature change can be dealt with more easily.

Thank you for your attention!

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