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# LAND OF THE CURIOUS

LES10A020 Engineering Physics

# LES10A020

# Engineering Physics
Lecture 9

by
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LUT University

# Revisiting the Ideal Gas Law

# Gas and Temperature: The Ideal Gas Law

- In case of gases, the relationship between temperature, pressure and volume are captured by ideal gas law, also called as general gas equation.
- In empirical form, it is: $\boxed{pV = nRT}$
- Here $p$ is pressure, $V$ is volume, $n$ is amount of substance, $T$ is temperature, and $R$ is the ideal gas constant:

$$R = 8.314510 \frac{\mathrm{Pa} \cdot \mathrm{m}^3}{\mathrm{mol} \cdot \mathrm{K}}$$

- Most gases follow this law quite well, main divergence occurring at extreme circumstances

# The Not-so Ideal Gas Law: Van der Waals

- The Van der Waals Equation of State offers an improvement to the ideal gas law
- Improvements:
  - the attractive forces between molecules, which are stronger at higher density and reduce the pressure, are considered by adding to the pressure a term equal to the square of the molar density multiplied by a positive coefficient $a$.
  - the volume of the molecules is represented by a positive constant $b$, which can be thought of as the volume of a mole of molecules.
- Resulting equation:

$$\left[p + a\left(\frac{n}{V}\right)^2\right](V - nb) = nRT$$

# PV-Diagrams

- While $T$ is constant (isotherm), the ratio between $P$ and $V$ changes.
- For ideal gas, they are inversely correlated.
- For Van der Waals gas, there is a strange region at low pressures and volumes.
- This non-ideal behavior needs to be death with.

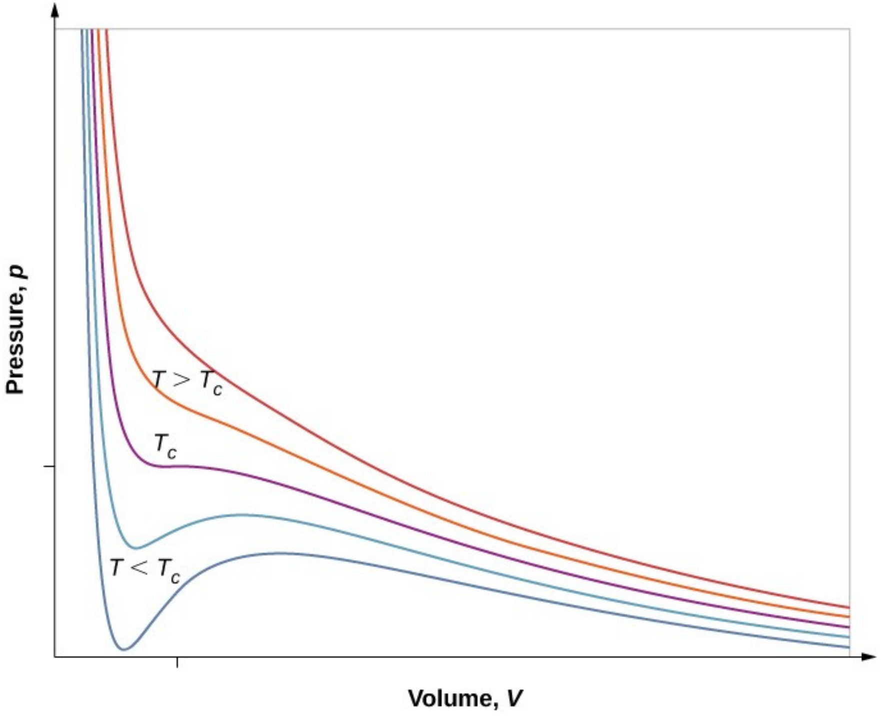
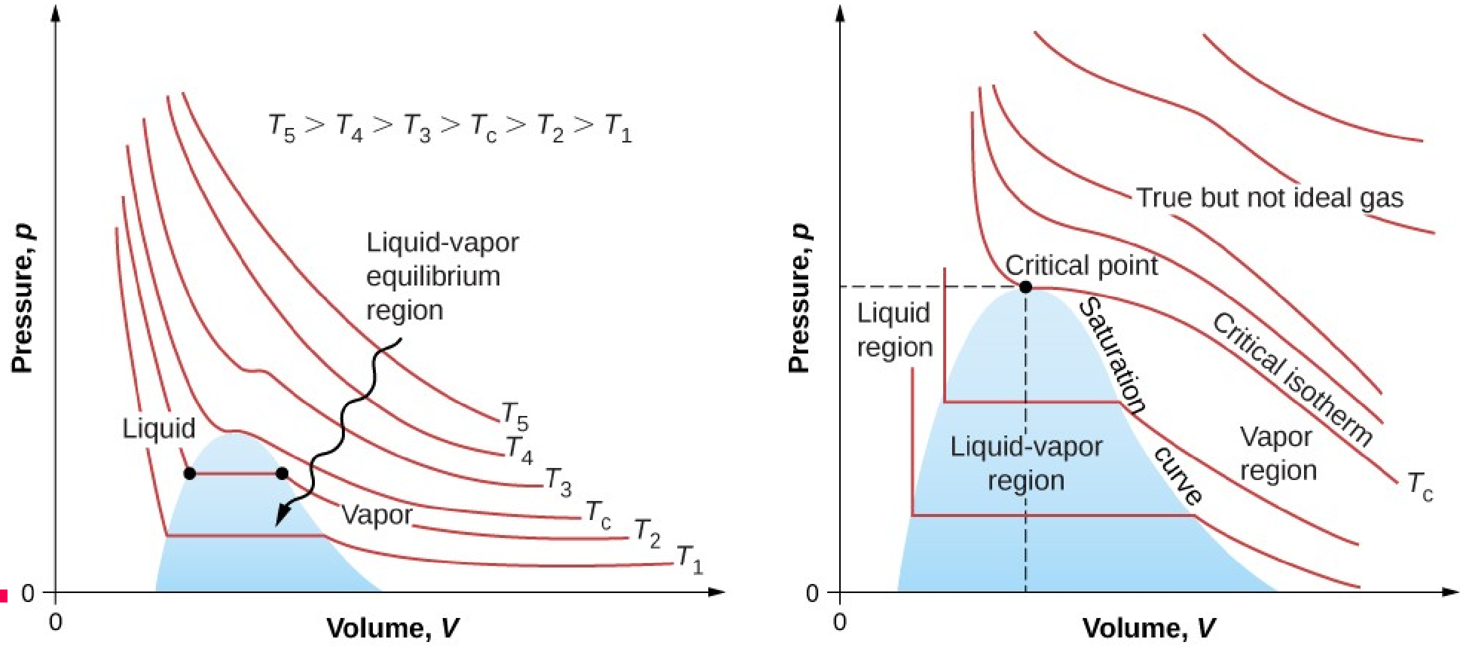


Figure source: Ling *et al.*, University Physics Volume 2, Ch. 2.1

# Critical Point Correction

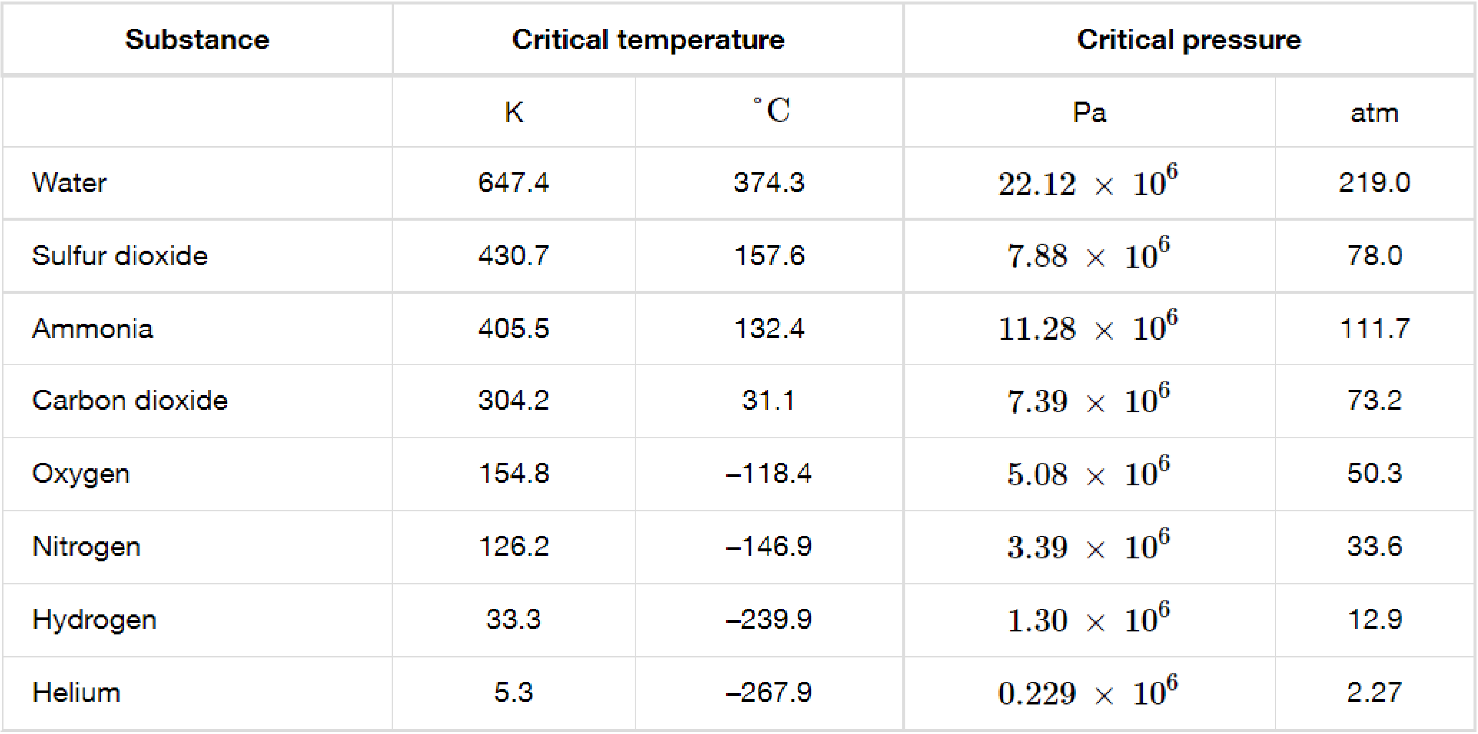
- The incorrect behavior is explained by liquid-gas phase transition region.
- If replaced by horizontal lines instead of simultaneous growth pressure and volume, a realistic image emerges.



(a) (b)

Figure source: Ling *et al.*, University Physics Volume 2, Ch. 2.1

# Critical Points of Various Substances



| Substance | Critical temperature | | Critical pressure | |
|---|---|---|---|---|
| | K | $^\circ\mathrm{C}$ | Pa | atm |
| Water | 647.4 | 374.3 | $22.12 \times 10^6$ | 219.0 |
| Sulfur dioxide | 430.7 | 157.6 | $7.88 \times 10^6$ | 78.0 |
| Ammonia | 405.5 | 132.4 | $11.28 \times 10^6$ | 111.7 |
| Carbon dioxide | 304.2 | 31.1 | $7.39 \times 10^6$ | 73.2 |
| Oxygen | 154.8 | –118.4 | $5.08 \times 10^6$ | 50.3 |
| Nitrogen | 126.2 | –146.9 | $3.39 \times 10^6$ | 33.6 |
| Hydrogen | 33.3 | –239.9 | $1.30 \times 10^6$ | 12.9 |
| Helium | 5.3 | –267.9 | $0.229 \times 10^6$ | 2.27 |

**Table 2.1** Critical Temperatures and Pressures for Various Substances

Table source: Ling *et al.*, University Physics Volume 2, Ch. 2.1

# Introducing Partial Pressure

- In a mixture of gases, the total pressure is the sum of partial pressures of the component gases, assuming ideal gas behavior and no chemical reactions between the components. This law is known as Dalton’s Law of partial pressures.
- Resulting from the ideal gas law and Dalton’s Law, we have

$$\frac{p_i}{n_i} = \text{constant}, \qquad \text{while also} \qquad p_{\text{total}} = \sum_i p_i$$

- Thus, any number of gases form the partial pressure that corresponds to the amount of that gas in the volume in question.
- The temperature and volume taken by a mixture of gases is always the same for all the gases.
- This knowledge can be applied when working with moisture of air.

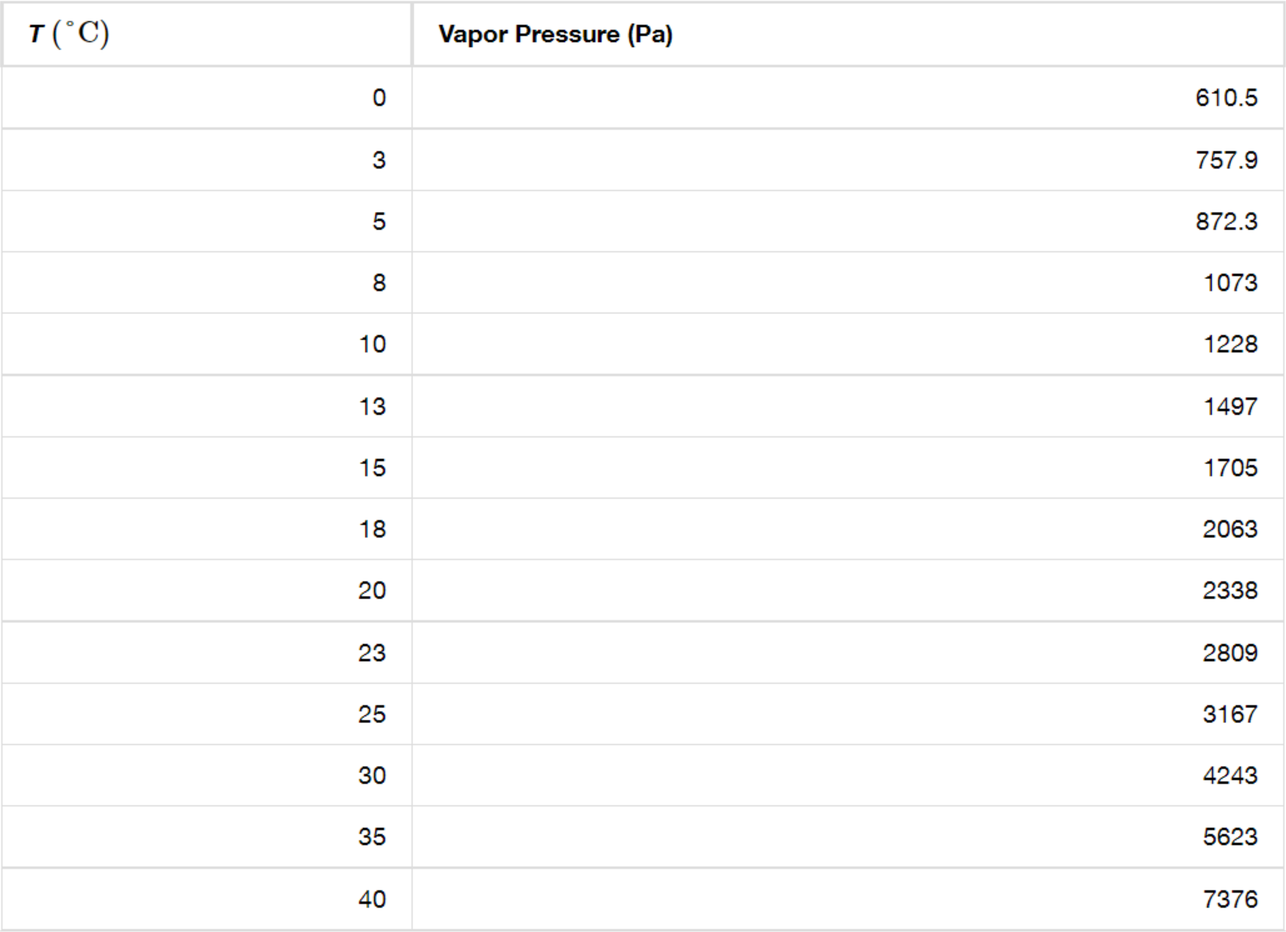
# Vapor Pressure

- Vapor pressure in the atmosphere is often discussed as the portion of the maximal amount of vapor on the air.
- For every temperature of air, there is a sharp limit how much vapor can be diluted into the atmosphere.
- The ratio of vapor in air and the maximum amount possible in that temperature is called relative humidity (R.H.)
- We can write it as

$$\mathrm{R.H.} = \frac{\text{partial pressure of water wapor at } T}{\text{full vapor pressure of water at } T} \times 100\%$$

# Vapor Pressure and Temperature

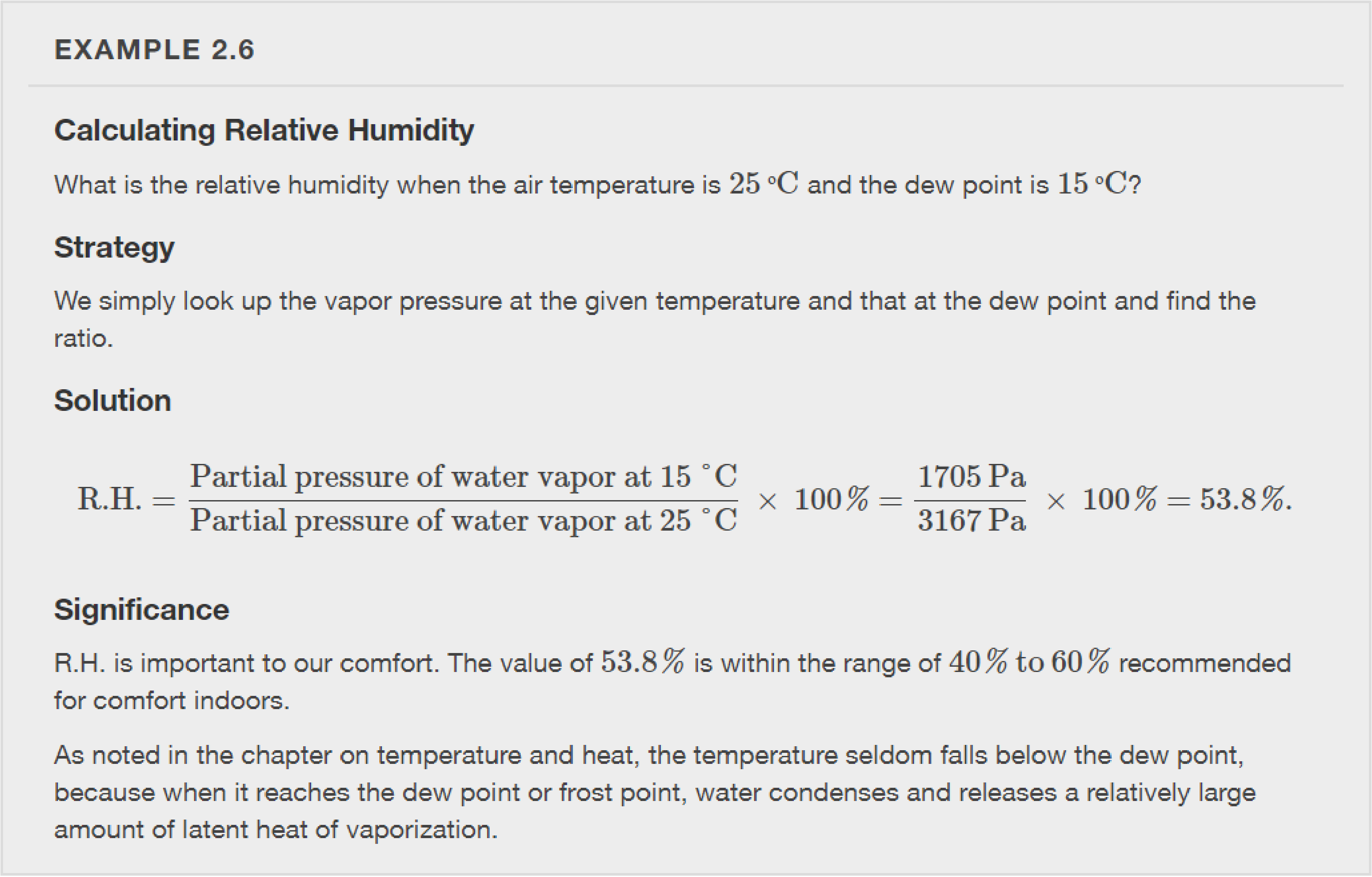
- This table show the vapor pressure as a function of temperature
- Pressure of 1 atm is assumed here.



| *T* (°C) | **Vapor Pressure (Pa)** |
|---:|---:|
| 0 | 610.5 |
| 3 | 757.9 |
| 5 | 872.3 |
| 8 | 1073 |
| 10 | 1228 |
| 13 | 1497 |
| 15 | 1705 |
| 18 | 2063 |
| 20 | 2338 |
| 23 | 2809 |
| 25 | 3167 |
| 30 | 4243 |
| 35 | 5623 |
| 40 | 7376 |

Table source: Ling *et al.*, University Physics Volume 2, Ch. 2.2

# Example: Relative Humidity



EXAMPLE 2.6

### Calculating Relative Humidity

What is the relative humidity when the air temperature is $25\ °\text{C}$ and the dew point is $15\ °\text{C}$?

**Strategy**

We simply look up the vapor pressure at the given temperature and that at the dew point and find the ratio.

**Solution**

$$\text{R.H.} = \frac{\text{Partial pressure of water vapor at } 15\ ^\circ\text{C}}{\text{Partial pressure of water vapor at } 25\ ^\circ\text{C}} \times 100\,\% = \frac{1705\ \text{Pa}}{3167\ \text{Pa}} \times 100\,\% = 53.8\,\%.$$

**Significance**

R.H. is important to our comfort. The value of $53.8\,\%$ is within the range of $40\,\%$ to $60\,\%$ recommended for comfort indoors.

As noted in the chapter on temperature and heat, the temperature seldom falls below the dew point, because when it reaches the dew point or frost point, water condenses and releases a relatively large amount of latent heat of vaporization.

Example source: Ling *et al.*, University Physics Volume 2, Ch. 2.2

# Revisiting Enthalpy and Internal Energy

- On Lecture 2, it was introduced that Enthalpy is the inner energy of the matter, when it only includes the energy resulting from heating the matter up:

$$H = U + pV$$

- In differential form, this can be expressed as

$$dH = dU + Vdp + pdV$$

- Recalling from before, we have for internal energy

$$dU = dW + dQ$$

- Recalling differential equation for work, we have

$$dW = \bar{\mathbf{F}} \cdot d\bar{\mathbf{s}}$$

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# Rewriting Enthalpy

- For gases, the force connects to pressure as $p = F/A$, while we can also consider that $A \cdot ds = dV$
- As a result, we can write for work $dW = -pdV$
- If we rewrite internal energy in differential form, we have

$$dU = dW + dQ$$

- If using for entropy $dS = dQ/T$, and for work $dW = -pdV$, it is possible to write for inner energy a new differential form:

$$dU = TdS - pdV$$

- Combing this to enthalpy, we can rewrite the equation as

$$dH = TdS + Vdp$$

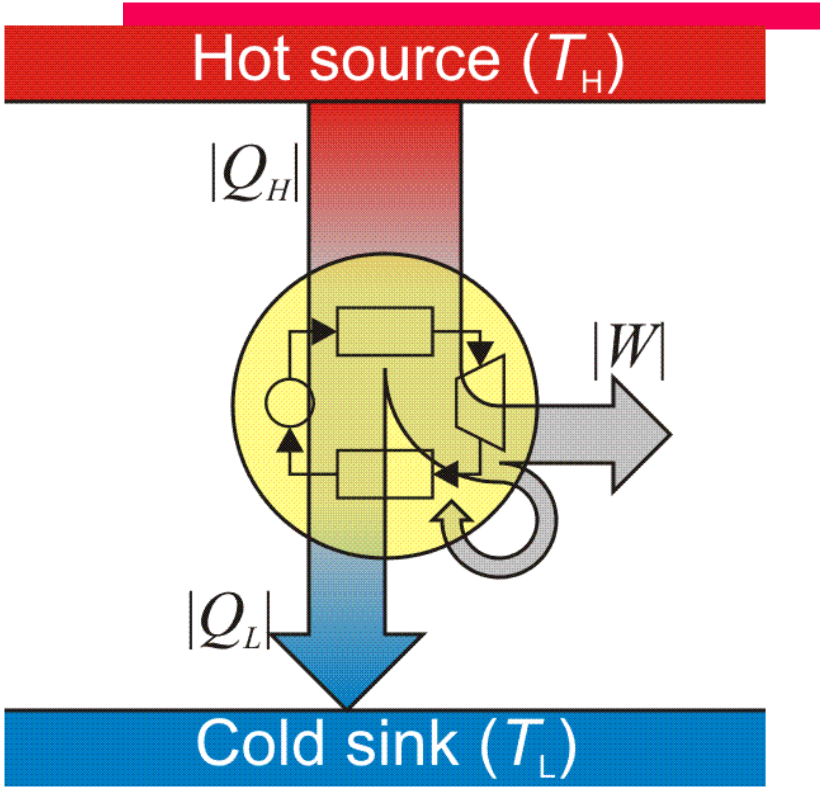
# Revisiting Heat Engine

# Heat Engine

- As we can recall, in Heat Engine part of the heat from a source $(Q_H)$ can be converted into work $W$
- Its efficiency is

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

- Here we can also replace $Q_i$ with $T_i$



# Inverse Heat Engine

- The heat engine can also be used in inverse (in figure, $Q_C = Q_L$) for refrigeration and heat pumps
- Now, different performance indicators are used for refrigeration (cooling) and heating using heat pumps
- For refrigeration, we have a coefficient of performance as

$$K_R = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

- For heating with a heat pump, the coefficient of performance is

$$K_P = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L}$$

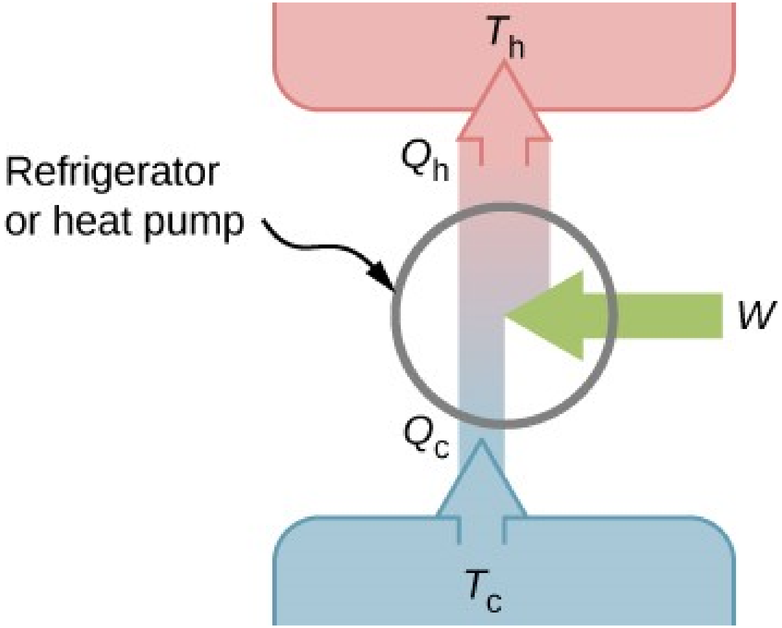


Figure source: Ling *et al.*, University Physics Volume 2, Ch. 4.3

# Collection of Equations

- A nice collection for the advanced equations of Thermodynamics is available online, at Libretexts.org
- See ”Differential Forms of Fundamental Equations” at https://chem.libretexts.org/@go/page/1933

Thank you for your attention!

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