BM20A8800 Discrete Models and Methods 3op

Exercise 4 / Week 6

1. We try to solve a 24-piece jigsaw puzzle (6 by 4 pieces, see example illustration on the right) by using trial and error without taking any advantage of the model picture – i.e. we don't know what the jigsaw picture represents. Let's say that the differences in jigsaw cutting pattern are not big enough to notice right away, so we don't know whether a piece will fit before we try it.

a) How many possible options are there – so, if we always have the worst luck with our selections, how many tries do we have to perform in order to get the puzzle done?

b) Now we identify that due to the manufacturing process there's a visible texture on the jigsaw: all pieces have horizontal lines. This information can be used to decrease our number of options. How many options are there now?

2. a) Define whether p and q are congruent to modulo m, when

i) $p = 134, q = 88, m = 6$ ii) $p = 1573, q = 12850, m = 7$

b) Pick a book from your bookshelf (if you don't have any books, go to library) and write down its author, name and ISBN. Then show following things:

i) The ISBN is correct. ii) Change 1 digit in the ISBN and show that now the check digit doesn't match.

Note: Choose a pre-2007 book so that it's in ISBN-10 form. If you choose a newer one, perform the calculations according to the ISBN-13 standard (see Wikipedia, for example).

3. a) Use the Euclidean algorithm in order to find out the greatest common divisor of m and n.

i) $m = 247, n = 117$ ii) $m = 1479, n = 272$

b) Find out (using gcd) whether the following equations are solvable in integers x and y. If yes, find out the solution using the extended Euclidean algorithm.

i) $3157x + 656y = 2173$ ii) $1054x - 833y = 2277$

4. Students of MVU University have freedom to pick their minor subjects as they wish – each has to select at least one, but the number of minors is not limited. Among students who started in 2020, 120 studied Applied Mathematics (AM), 200 Computer Science (CS) and 60 Advanced Statistics (AS). 40 of these students studied both CS & AS, 80 both AM & CS and 30 both AM & AS. 20 of these students were multitalented (or just optimistic with their time resources) and had picked all three minors.

a) How many students are there in total in this "year of 2020" class in MVU?
b) Illustrate the situation using a Venn diagram, where you calculate all the the "areas". Check your answer to a-section by summing up all these areas.

5. a) In the beginning of 21st century, UEFA Champions League (football, if you didn't know) was arranged among the participating 32 teams in the following way, before it was ruined:

In first group stage teams were divided to eight groups, where each team played twice against each other (in home and away games). After this, the two best teams from each group advanced to second group stage, where teams were divided to four groups. In these groups teams again played twice against each other. Two best teams of each group advance to playoffs, where teams meet in 1-on-1 matchups. Matchup teams play home and away games against each other and the result is based on the combined result of these 2 games. Winner advances to next playoff round, loser drops out from the competition.

How many games are played in total during the whole season? (Document your solution and explain how many games are played in each stage – just a plain answer is not enough!)

b) In traditional poker (not Texas Hold'em or other fancy new garbage) each player is dealt five cards ("hand"). Players are allowed to then replace "bad" cards by drawing new cards from the deck and try to improve their hand. One hand that is considered fairly good is "full house" – so, a combination of three cards of same number and two cards of (naturally some other) same number. (Example: 3 jacks and 2 aces.) If a traditional 52-card deck* is used (so, without jokers), how many different full house combinations exist? (Again, document your solution; don't just search the number from internet.)

*If you're unaware of what a deck of cards consists of, please find out from Wikipedia or such.

6. a) Petri plays Scrabble with his friend from abroad. They have agreed that Petri can formulate Finnish words and his friend can formulate English words. In the end of the game, Petri notices that he's in a bad situation: he'd have to figure out at least a six-letter word out of his letters RAARALL. Luckily Petri is from the region of Savo, so he is able to invent an explanation for any possible "word" he forms. Considering future games, he anyhow tries to come up with a solution that is as believable as possible, leaving his friend with as little doubt of foul play as possible. How many different "word" options does he have, so that he can evaluate what's the best?

b) Aki is in a student party where drinks from the tap are sold in pints for a fixed price of 4 euros. Tap drinks include beer, apple cider, pear cider and long drink. Aki's party posse consists of him and 4 other persons, and the posse orders drinks in rounds where each member of the posse pays a round on their turn. Aki is a bit low on cash, so he tries to sneak out before it's his turn, but fails. Luckily Aki has 20 euros in his pocket, so he can afford one round of drinks for all. Unfortunately he forgets to ask what each member wants to drink, so he has to make the decision on the counter under pressure, when the DJ plays ZZ Top. How many different pint combinations can he bring to the table?

Answers/hints for selected problems:

1. a) Little over $3 \cdot 10^{19}$ *b) Just a little under* $4 \cdot 10^{13}$

(Hint: draw a picture in grid paper and notice that the pieces can be divided into 3 different categories.)

2 & 3: you can check your solutions using internet calculators or Matlab (or by substitution in 3b).

4. a) 250

5. a) 158 games b) 3744

6. a) 420 b) –