1. a) 3 frames:

- LRU replacement: 18 page faults

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ref		7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
f		7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
f			7	2	3	1	2	5	3	4	6	6	7	1	0	5	4	6	2	3	0
f				7	2	3	1	2	5	3	4	4	6	7	1	0	5	4	6	2	3
hit		✗	✗	✗	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
v					7		3	1	2	5	3		4	6	7	1		5	4	6	2

- FIFO replacement: 17 page faults

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ref		7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
f		7	2	3	1	1	5	5	4	6	7	7	1	0	5	4	6	2	3	0	1
f			7	2	3	3	1	1	5	4	6	6	7	1	0	5	4	6	2	3	0
f				7	2	2	3	3	1	5	4	4	6	7	1	0	5	4	6	2	3
hit		✗	✗	✗	✗	✓	✗	✓	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
v					7		2		3	1	5		4	6	7	1		5	4	6	2

- Optimal replacement: 13 page faults

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ref		7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
f		7	2	3	1	1	5	5	4	6	7	7	7	0	0	4	6	2	3	3	3
f			7	2	3	3	1	1	5	5	5	5	5	5	5	0	0	0	0	0	0
f				7	2	2	3	3	1	1	1	1	1	1	1	1	1	1	1	1	1
hit		✗	✗	✗	✗	✓	✗	✓	✗	✗	✗	✓	✓	✗	✓	✗	✗	✗	✗	✓	✓
v					7		2		3	4	6			7		5	4	6	2		

1. b) 4 frames:

- LRU replacement: 17 page faults

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ref		7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
f		7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
f			7	2	3	1	2	5	3	4	6	6	7	1	0	5	4	6	2	3	0
f				7	2	3	1	2	5	3	4	4	6	7	1	0	5	4	6	2	3
f					7	7	3	1	2	5	3	3	4	6	7	1	0	5	4	6	2
hit		✗	✗	✗	✗	✓	✗	✓	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
v							7		1	2	5		3	4	6	7	1		5	4	6

- FIFO replacement: 17 page faults

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ref		7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
f		7	2	3	1	1	5	5	4	6	7	7	1	0	5	4	6	2	3	0	1
f			7	2	3	3	1	1	5	4	6	6	7	1	0	5	4	6	2	3	0
f				7	2	2	3	3	1	5	4	4	6	7	1	0	5	4	6	2	3
f					7	7	2	2	3	1	5	5	4	6	7	1	0	5	4	6	2
hit		✗	✗	✗	✗	✓	✗	✓	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
v							7		2	3	1		5	4	6	7	1		5	4	6

- Optimal replacement: 11 page faults

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ref		7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
f		7	2	3	1	1	5	5	4	6	6	6	6	0	0	4	4	2	3	3	3
f			7	2	3	3	1	1	5	5	5	5	5	6	6	0	0	0	0	0	0
f				7	2	2	3	3	1	1	1	1	1	5	5	6	6	6	6	6	6
f					7	7	7	7	7	7	7	7	7	1	1	1	1	1	1	1	1
hit		✗	✗	✗	✗	✓	✗	✓	✗	✗	✓	✓	✓	✗	✓	✗	✓	✗	✗	✓	✓
v							2		3	4				7		5		4	2		

2.

Virtual address	Binary	Page	Frame	Physical address
0xE12C	1110 0001 0010 1100	14	3	0x312C
0x3A9D	0011 1010 1001 1101	3	10	0xAA9D
0xA9D9	1010 1001 1101 1001	10	5	0x59D9
0x7001	0111 0000 0000 0001	7	15	0xF001
0xACA1	1010 1100 1010 0001	10	5	0x5CA1

3. $EAT = (1 - p) \times Memory\ Access\ Time + p \times Page\ Fault\ Service\ Time$

EAT = 200 nanoseconds = 0.0002 milliseconds

MAT = 100 nanoseconds = 0.0001 milliseconds

$$\Leftrightarrow 0.0002 = (1-p) \times 0.0001 + p \times (0.7 \times 20 + 0.3 \times 8)$$

$\Longrightarrow p = \frac{1}{163999} \approx 0.000006$

4. a)

i) The initial value of the counters is 0

ii) The counters are increased whenever a new page is associated with that frame

iii) The counters are decreased whenever one of the pages associated with that frame is no longer needed

iv) A page to be replaced is selected as the frame with the smallest counter, with a FIFO queue to break ties

4. b) 14 page faults

4. c) 11 page faults