P1

Math A : Assignment one

1. $A=\{1,2,3\}$, $B=\{1,3\}$.

$P(A)=\{\phi, A, \{1\}, \{2\}, \{3\}, \{1,2\} \{1,3\}, \{2,3\}\}$

$P(B)=\{\phi, B, \{1\}, \{3\}\}$

$m(A)=3$, $m(B)=2$.

$m(P(A))=2^3$. $m(P(B))=2^2$.

$\Rightarrow$ if $m(X)=n$, then $m(P(X))=2^n$.

$A-B=\{2\}$,

$P(A-B)=\{\phi, A-B\}$.

p2

$P(A) - P(B) = \{ \{2\}, \{1,2\}, \cancel{\{1,3\}}, \{2,3\}, A \}$

$P(\{\phi\}) = \{ \{\phi\}, \{\phi\} \} = \{ \{\phi\} \}.$

error: $\{\phi\}$ is changed by ϕ. then $P(\{\phi\}) = \{ \phi, \{\phi\} \}.$

2. $X = \{ x^2 \mid x \in \mathbb{Z} \}$

$\hat{X} = \{ x \in \mathbb{R} \mid x < 1 \} \boxed{= \{ x < 1 \mid x \in \mathbb{R} \}}$ ← forget about this notation.

3. $A = \{1,2,3,4\}, \quad B = \{1,2\},$

$A \times B = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2)\}.$

$B \times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)\}.$

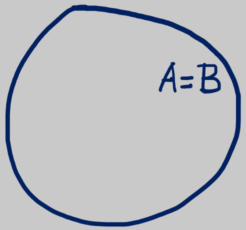
$A \times B - B \times A = \{ (3,1), (3,2), (4,1), (4,2) \}.$

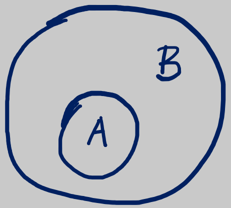
4. Let A, B and C be subsets of universe U.

(a) $(A \cup B) \subseteq B$.

From $B \subseteq A \cup B$, it follows that

$A \cup B = B$, then $A \subseteq B$.





(b). $A \subseteq B$, $A \subseteq C$, $(B \cap C) \subseteq A$.

From $A \subseteq B$ and $A \subseteq C$,

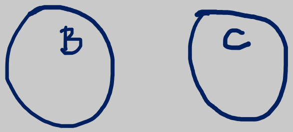
we know that

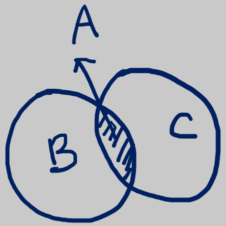
$\forall a \in A$, $a \in B \cap C$,

this gives $A \subseteq B \cap C$.

Together with $(B \cap C) \subseteq A$,

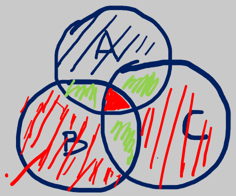
we find $A = B \cap C$.

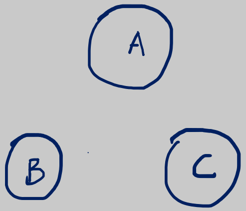


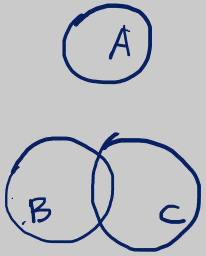


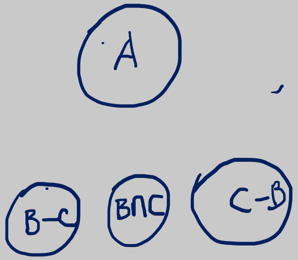
P4.

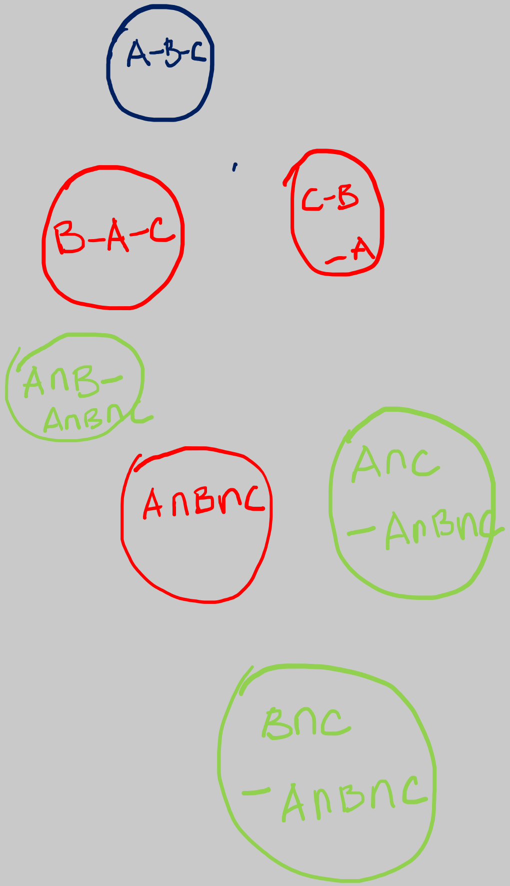
5. (a). $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

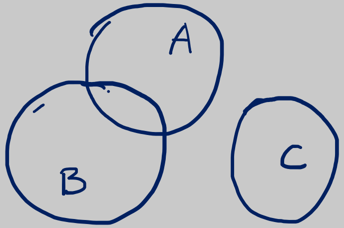
④.  $\Leftrightarrow$

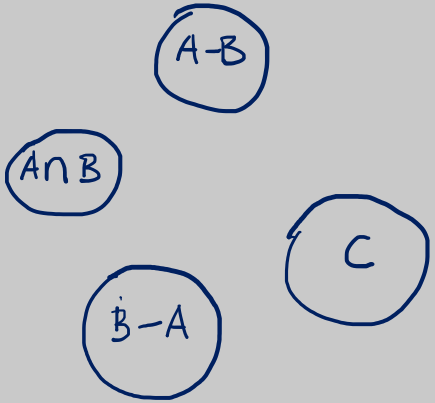
①. 

②. 

$\Leftrightarrow$ 



③. 

$\Leftrightarrow$ 

P5

5. (a). $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.

$A \cap (B \cup C) \supseteq (A \cap B) \cup (A \cap C)$

proof $\Rightarrow$ If $\forall\, a \in A \cap (B \cup C)$, then $a \in A$ and $[a \in B$ or $a \in C]$,

which is to say $[a \in A$ and $a \in B]$ or $[a \in A$ and $a \in C]$,

Then we rewrite the relation by

$a \in (A \cap B \quad \cup \quad a \in A \cap C$

$\Leftarrow$ If $\forall\, a \in (A \cap B) \cup (A \cap C)$, then $a \in (A \cap B)$ or $a \in A \cap C$. This implies that $[a \in A$ and $a \in B]$ or $[a \in A$ and $a \in C]$, which is

$a \in A$ and $[a \in B$ or $a \in C]$.

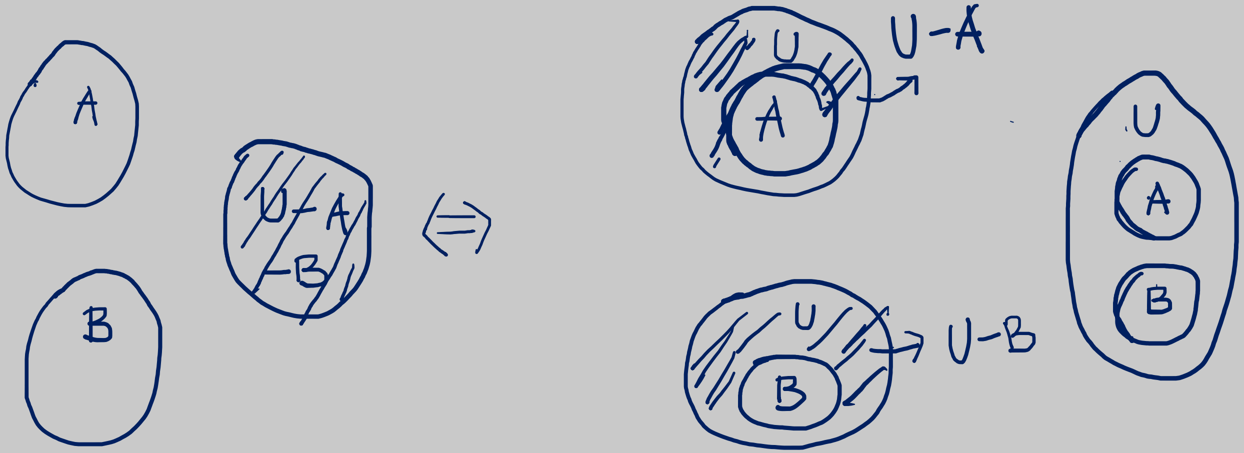
P6.

6. (a). A, B are two generic sets, Prove

$$(A\cap B)^c = A^c \cap B^c.$$

proof: $(A\cap B)^c \subseteq A^c \cap B^c$

$(A\cap B)^c \supseteq A^c \cap B^c.$



$U-A-B$

$= (U-A) \cup (U-B)$

$= (U-A-B) \cup (A\cap B)$

$\Rightarrow$: $\forall\, a \in (A \cap B)^c$, $a \notin A \cap B$, which is $a \notin A$ $\boxed{\text{or}}$ $a \notin B$

$a \in A^c$ or $a \in B^c$. This is to say $a \in A^c \cup B^c$.

$\Leftarrow$: If $\forall\, a \in A^c \cup B^c$, then $a \in A^c$ or $a \in B^c$,

$a \notin A$ or $a \notin B$. This means $a \notin A \cap B$ or

$a \notin A \cap B$. Then $a \in (A \cap B)^c$.